MUSICAL ANALYSIS AND THE REPRESENTATION

Within the course of its history, musical analysis developed a great number of representations with little interest on the principles and aims of representing. Representation in music is facing a double difficulty: the temporal nature of this art, and the fact that acoustic phenomena are nearly impossible to see. Nevertheless, most of the representations used by musical analysis, and first of all musical notation itself, deal with visual space. Despite of that, the representations of music used for analysis purposed are far from being of the same nature. They can be symbolic, graphic or even, more recently, animated figures on multimedia support. How does the transfer from the sonic domain to its representation operate? This is a great question. But it underlies another one, dealing with the problematic nature of representation. Music is a phenomenon implying the presence of several subjects through what is usually considered as an object, even if this word is specially unclear concerning music. This means that musical representation can not avoid to have a mental aspect, dealing with psychological and socio-cultural maters. How can musical analysis account for that aspect? Furthermore, any representation of a phenomena induces probably a modification in our way of perceiving it. Representation is then as much a testimony on reality than a way to filter or even to deform it.

Even if it seems difficult to avoid a typology of the traditional representations used in the field of musical analysis, a special emphasis will be placed on the recent evolution (or introduction) of some new ways of representing music. Topologic problems aroused from generalisation of Euler's nets will be particularly emphasised, as they are prototypic of what we will call *representational spaces*. Harmonic organisation suggest a multiplicity of distances, and then a competition within different kind of representations witch can take into account different aspects of musical organisation. In this idea, there is not one best representation, but a representation more appropriate to show certain properties. The variety of possible Tonnetz, and their ability to show important aspects of harmonic and melodic systems will be demonstrated.

But representation of music can not consist only in a *map* of relations between predefined concepts called "notes". One of the main topic about music representation is the way it is able to deal with the *dynamics* aspects of the phenomenon, that is to say with its temporal behaviour. One of the more evident way to comply this goal is to represent a trajectory in the representational spaces already mentioned. This idea can lead to a generalisation of the concept of "phases spaces" initiated by Poincaré. The equivalence of phase space representation and structural grammar is a very important result, as it means there is a continuity in between physical and symbolic representation. It is also important to understand how this overall conception of representation can be realised from the acoustic signal itself. The concept of DFT (Differential Fourier Transform) is then very important as well as a conception of representations as projections of an overall scheme implying representational space, time and variation space.

Representation is not the aim of musical analysis, but it is a very important clue to understand its methodological procedures. Even if some refined spirits will say that the best analysis is the music itself, we must consider the fact that all known practises of musical analysis have something to do with a representation. Those representations are of different kinds, according to the aspect of music that is focused and to the specific theoretical context the analysis is related to. Until the middle of the twentieth century the analytical theories where mainly concerned by a symbolic representation of music, even if the musical notation itself has a very ambiguous intermediate position in between symbolic and geometric considerations. The attempt by Rousseau to generalise the numeric representation initiated by some old tablature notations and renewed in the context of the birth of commercial musical computers by the MIDI system, evidences the possibility of a transfer from music to the world of arithmetic, taking into account a subjacent theory of order in specific musical scales. This way of seeing was further enhanced to fit the ideas of serial music and generalised through Forte's *set theory*. One could consider the work of Juan Carrillo to fit the same premises for microtonal scales. The use of numbers in harmonic notation of the chords can be directly connected to a theory of harmony further completed by the works of Rameau on fundamental bass and the work of Riemann on harmonic functions. Those theories tend to extract the purpose of the representation from the notation itself to an overall way of seeing the musical system. This is clearly the aim of Schenkerien analysis, even if it seems to be an analytic method that proposes a representation that seems to remain very close to the musical notation itself.

As we said, the traditional musical notation can also be understood in a geometrical way. Music seems to be eligible for a representation in Cartesian spaces. This means that it can be understood as the evolution of a phenomenon within the time. It also means that it is possible to describe this phenomenon with specific parameters and that it has some meaning to assimilate the time to a spatial axis… The *piano roll* is the first example of this possibility. Nevertheless the so-called Cartesian representation has been for centuries the preferred representation for scientific demonstration, and it has been of course very useful for musical analysis, in so many ways that it is impossible to quote them here. The main change proposed by scientific representation for musical analysis purposes is probably the "sonogram", anticipated by Helmoltz, but becoming technologically affordable only in the middle of the twentieth century. This is because it allowed to represent the entire world of sound and not only the formalised selection proposed by musical tradition and eligible for symbolic representation. The sonogram is a clear example of a representation that is not related to musical theory, but to a theory (Fourier's analysis) far underneath any musical consideration.

1. Representing the music or representing its analysis?

This leads us to a specific statement: analysis needs a representation before its own process, and a representation of its results, and those representations are not necessarily the same. Let's have a look to an interesting epistemological example, taken from Hans Mersmann's *Versuch einer Phänomenologie der Musik*[1]. In this publication, dating from 1922, Mersmann intends to fill the gap between the scientific components of art theory and the specific substance of æsthetic for which notions like "true" or "false" does not mean anything. In his proposal, musical analysis is intending to articulate a relation in between form and content. We will focus here the very interesting way Mersmann represent his ideas, about Haydn's piano sonata Hob.XVI:49 :

Fig. 1: First page of the first edition of Haydn's Piano Sonata No.59 Hob.XVI:49, E-flat major (Vienna, Artaria, n.d. [1791]) <http://imslp.org/wiki/Special:ImagefromIndex/52562>.

Hans Mersmann uses the well known shaping of the phrases by antecedent [*Vordersass*] and result [*Nachsass*], but the interesting point for us is the way he represents this shape on a unique diagram of the whole piece, according to the "motivic forces" [*motivische Kräfte*] and a vertical axis assimilated to a "tension" parameter [*Spannung/Entsannung*].

[1] Hans Mersmann, « Versuch einer Phänomenologie der Musik », Zeitschrift für Musikwissenschaft. 5 (1922/23), S. 226-269, reprint Wissenschafliche Buchgesellschaft, Darmstadt, 1973.

Fig. 2: Analysis of Haydn's Piano Sonata No.59 Hob.XVI:49, E-flat major *proposed by Hans Mersmann (op. cit. p. 31).*

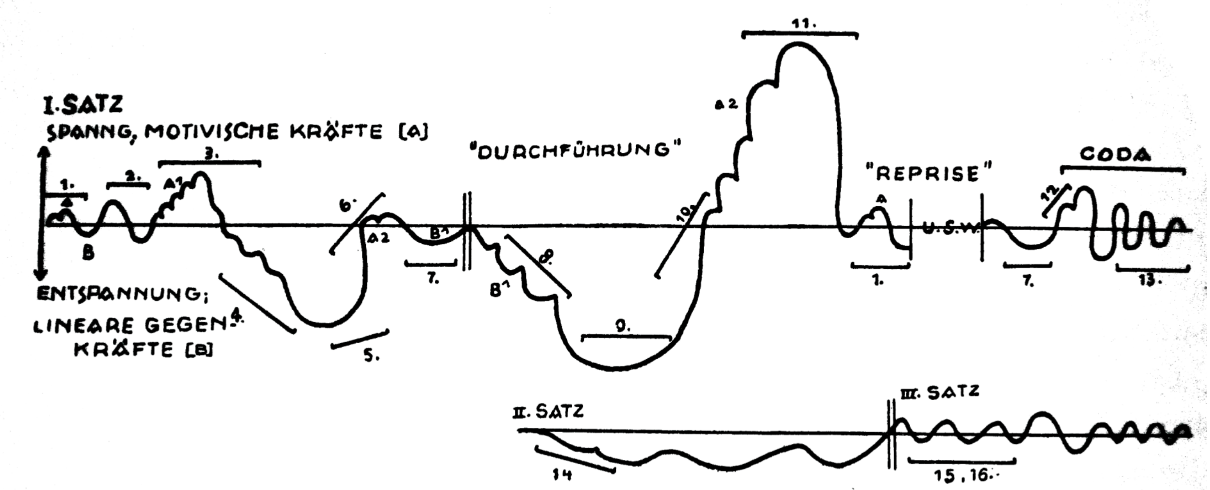

*Fig. 3: The resulting diagram proposed by Hans Mersmann, with tension [*Spannung/Entsannung*] axis (op. cit. p. 36).*

Further on in his paper, Hans Mersmann gives an other astonishing diagram, trying to provide an overall idea of the morphologic evolution of western music.

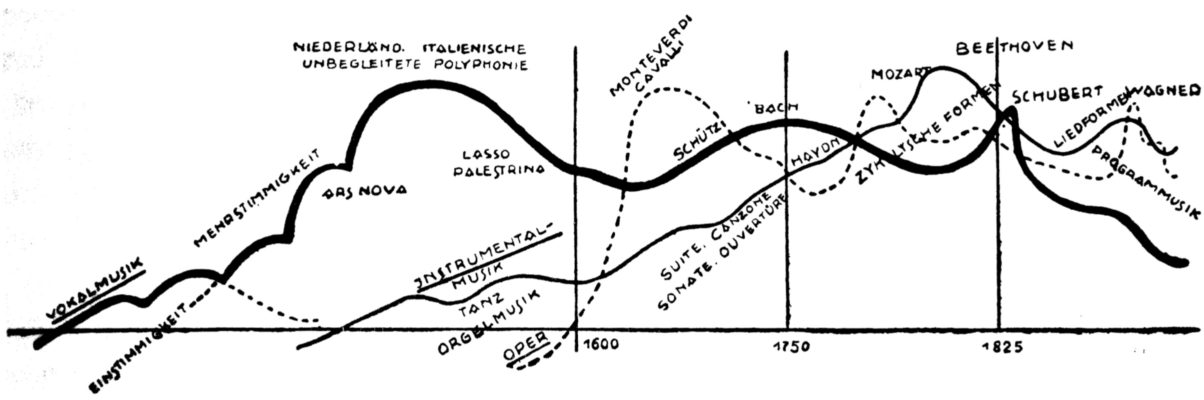

Fig. 4: Hans Mersmann, an interpretation of his former representation about western music history (op. cit. p. 39).

This drawing is presented as a phenomenological construction. It is nevertheless realised as if it was a scientific evidence, using the frame of standard Cartesian axis. But the "parameter" represented on the vertical axis is a construction that supposes an overall judgement on all the music produced during several centuries. It is by no way possible to imagine a specific algorithm that would process such an amount of data and give a result that could fit the aims of the representation, first by finding the proposed categories. Such an example tells us what is the difficulty for musical analysis: dealing with a projection of the real world in an equivalent that is not too far from the original and that allows to show the specific relations that are looked for, and then giving a projection of those results in a semantic space meaningful for the music analysed. We propose to call those two moments of the analytic process descriptive and interpretative representations. This conceptual separation is indubitably useful, but of course, the purpose of a fair analytical process is to be sufficiently transparent so that both representations can be related the simplest and the closest way possible.

This means analysis is always very concerned by the way each step derives one from the other. This is the real meaning of re-presenting: building a reliable duplication that catch the reality on a specific perspective, without modifying its characteristics. Whenever a representational process acts as a filter, a non-linear deformation, etc. it means that the representation looses or modifies the initial information. This kind of process can be very useful for analytic purpose, and, in some way, "Shenkerian like" analysis (e.g. the *dynagrams* proposed by Jörg Langner[2]) is a kind of progressive filtering of the information that leads to the inner structure of a given work. But it is far more meaningful if the entire process is shown off than if the (a little bit obvious) last step is presented on his one.

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2 Jörg Langner, *Multidimentional Dynamic Shaping*, Proceedings of the Third Triennial ESCOM Conference, Uppsala, 1997, pp. 713-718. Also : Jörg Langner, Reinhard Kopiez, Martin Wilz, « realtime analysis of dynamic shaping » <http://www.youscribe.com/catalogue/tous/art-musique-et-cinema/autres/realtime-analysis-of-dynamic-shaping-jorg-langner-humboldt-427995>

The case of Langner's representations is very interesting, as it shows not the music itself, nor the result of an analysis, but the gradual process involved, and its results are more interesting when comparing several interpretations of the same work, witch means that it is able to grab very useful considerations about the behaviour of some under evaluated parameters such as dynamics, and the way they give shape to the overall structure of a work.

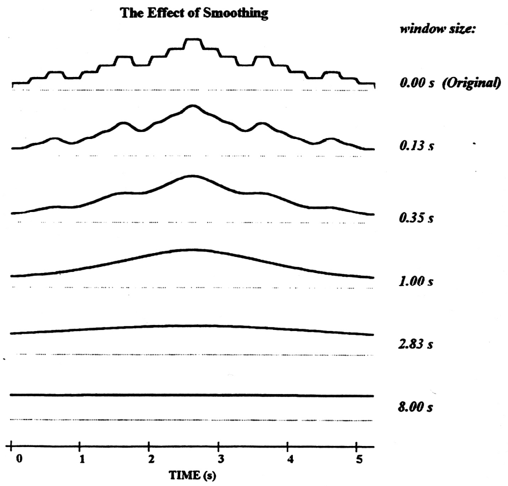


Fig. 5: "The effect of smoothing. The original curve is similar to the loudness curve of [former figure] (merely the gaps between the onsets are filled). Smoothing with a window size of 1.00 seconds leads to a curve showing purely the one wide arc of the original. Smoothing with a window size greater than the duration of the example results in a horizontal line representing the mean value of the whole curve". (Jörg Langner, 1997, op. cit. p. 713-718)

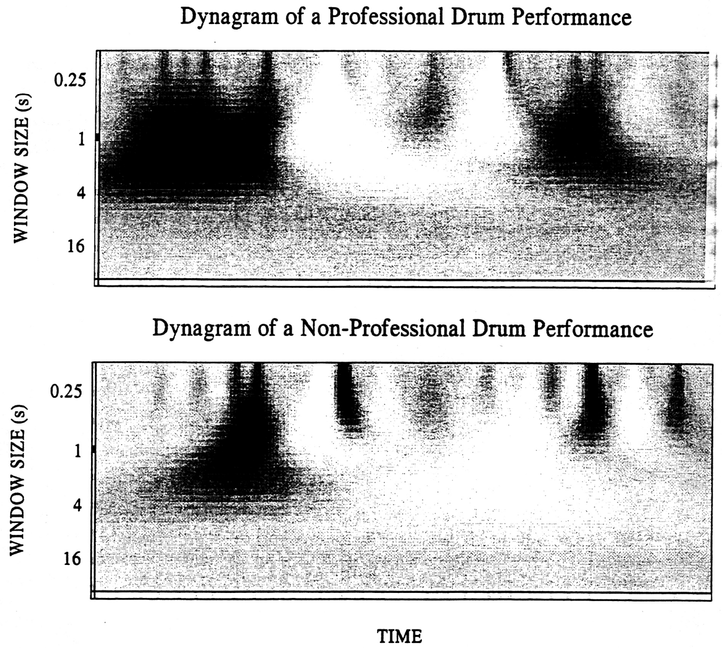


Fig. 6: "Dynagrams of a professional and a non-professional drum performance of the rhythm notated [in the former figure] (dark shading: crescendo; middle gray: constant loudness; light shading: decrescendo). We can see the traces of the stronger and more extensive dynamic shaping of the professional. Note that the horizontal axis in these figures show no markings for the timeunit (seconds) but for the onsets of the notes". (Jörg Langner, 1997, op. cit. p. 713-718

Mersmann and Langner give us an interesting idea of what a single parameter is able to generate as time complexity. But it seems difficult to accept the idea that music can be summarized with a single parameter, even if this parameter is supposed to reach a kind of abstraction of the overall effect of the piece in the listener's perception, like the "tension" parameter is supposed to do. This leads of course to important questions about the purpose of analysis itself, and the way an "objective" representation can be related to a "subjective" grid of understanding.

2. Representational spaces, topology and projections.

The case of an unique parameter for musical representation is very reducing. It seems accurate for pitch or dynamic, as we have seen, but by no way sufficient for timbre or harmony. Therefore a lot of representations have been proposed for those dimensions of musical experience. It would be impossible to quote all those representations here, we will just have a look on two of them, published in *Science*, showing the renewed interest those last years for this problems, and the kind of complexity multidimensional representations can represent.

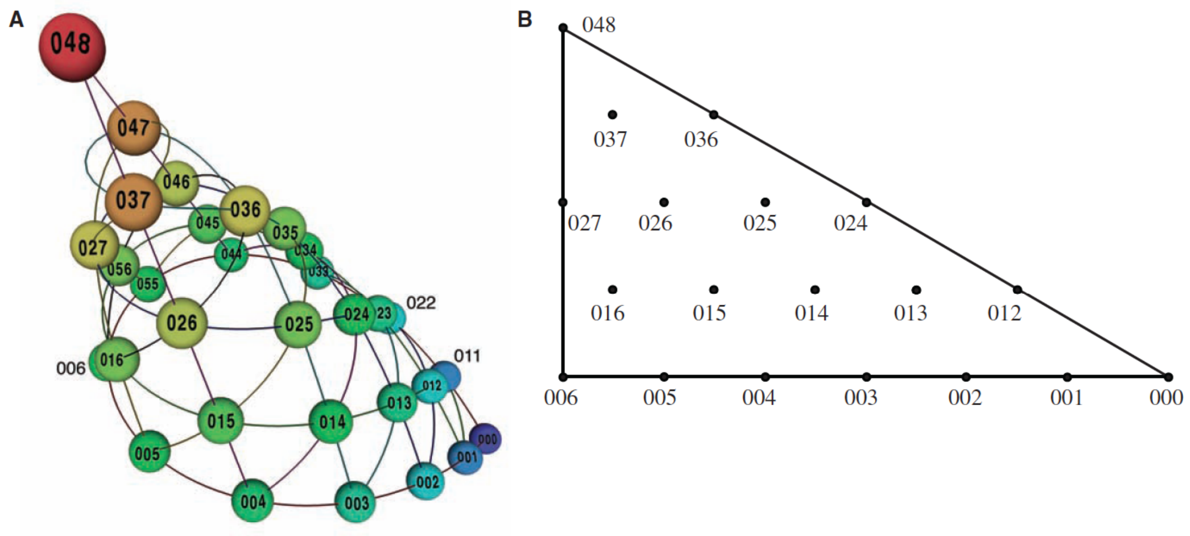


Fig. 2. **(A)** $\mathbb{T}^2/\mathcal{S}_3$ is a cone. **(B)** $\mathbb{T}^2/(\mathcal{S}_3 \times \mathbb{Z}_2)$ is a triangle. Numbers refer to pitch classes, with 0 = C, 1 = C♯, etc. Points represent equivalence classes of transpositionally (A) or transpositionally and inversionally (B) related chords. Thus, (C, D, E) and (D, E, F♯) are both instances of 024.

Fig. 7: Representation of the basic 3 sound chords taking into account the several possible symmetries[3]

[3] Clifton Callender, *et al.*, "Generalized Voice-Leading Spaces", *Science* 320, 346 (2008)

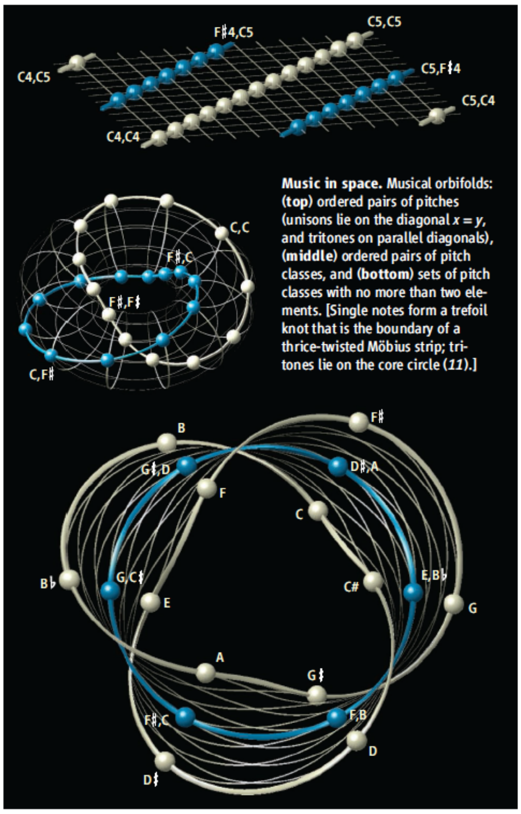


Fig. 8: Representation of a Tonnetz with a tore and a Möbius stip [4]

Several intends have been made to take into account the fact that a great number of symmetries and cyclicities are possible. It comes very quickly to a saturation of the common bi-dimensional possibilities of a paper or a screen. The representation proposed by Gilles Baroin in its "Planetes-4D" "model", intending to give account of the cycles of major and minor cycles at the same time is a good image of the problem to solve, and of the difficulty to mentalise a multi-dimensional space.

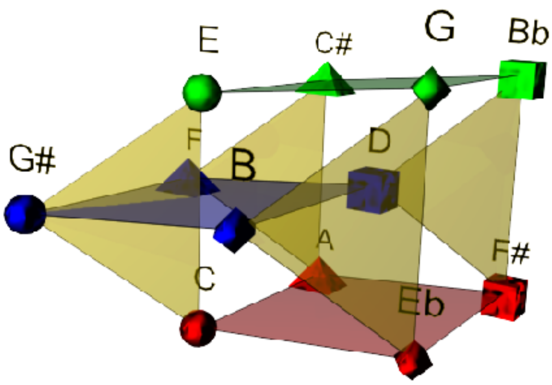
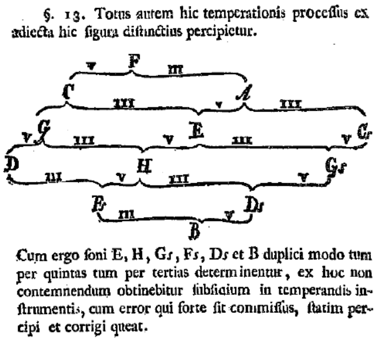


Fig. 9: A 3D projection of the model 'Planetes-4D' from Gilles Baroin[5]

[4] Rachel Wells Hall, "Geometrical Music Theory", *Science* 320, 328 (2008)

[5] Gilles Baroin , « De Newton à Riemann, Graphes et Graphisme : Interactions Mathématico-Musico-Plastiques », *Litter@incognita*, n°3 (2010) - Numéro 2010, p. 41 – 57, mis en ligne le 15/11/2010. <http://e-revues.pum.univ-tlse2.fr/1/littera-incognita/article.xsp?numero=3&id_article=article-5-baroin-1072> (Consulté le 26/09/2011).

The obvious problem of those representations is that they are very interesting for mathematicians but not so meaningful for musicians and thus not very helpful for musical analysis. An effort is to be made to understand the criterions for a compatibility with musical understanding of those kind of representations, especially in the context of harmonic representation. The specific representation, initiated by Euler in the eighteenth century, and further developed by Riemann and the "neo-riemanian" school in the late twentieth century will provide a very interesting example of the epistemological difficulties musicologists are confronted to.



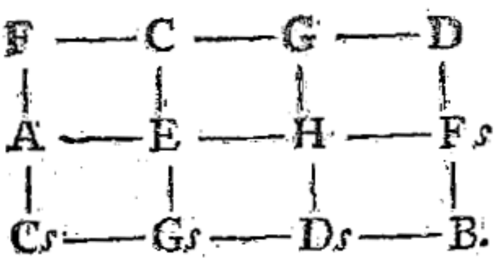


Fig. 10a: Tonnetz proposed by Euler in his Tentamen novae theoriae musicae ex certissismis harmoniae principiis dilucide expositae*, 1739, p. 147.*[6]

Fig. 10b : Tonnetz proposed by Euler in his De harmoniae veris principiis perspeculum musicum repraesentatis *original publication in:* Novi Commentarii academiae scientiarum Petropolitanae *18, 1774, p. 350.*[7]

The problem that made this kind of representation emerge is interesting. It is related to the cyclic nature of the intervals in a twelve tones scale. In Euler's figure, it is the confrontation in between fifth (V on the first diagram) and major thirds (III on the first diagram). His problem is about temperament, and the impossibility to have a perfect octave summing 12 perfect fifth or 3 perfect major thirds.[8] This is precisely what allows equal temperament. It is quite obvious that all intervals can pretend to build such a paving, and then, the question of choosing the more "meaningfull" one is an important question, perhaps not for mathematicians, but clearly for musicologists.

Richard Cohn intended to give some mathematical clues on that question, using the idea of "parsimonious" voice leading, and the three "PLR" operations defined by Riemann and formalised by David Lewin, P for Parallel, L for Leading-tone and R for Relative.

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[6] http://math.dartmouth.edu/~euler/docs/originals/E033p80-168.pdf (consulté en mars 2009)

[7] http://math.dartmouth.edu/~euler/docs/originals/E457.pdf (consulté en mars 2009)

[8] The use of this diagram by Riemann, more than a century later, is related to the same problem.

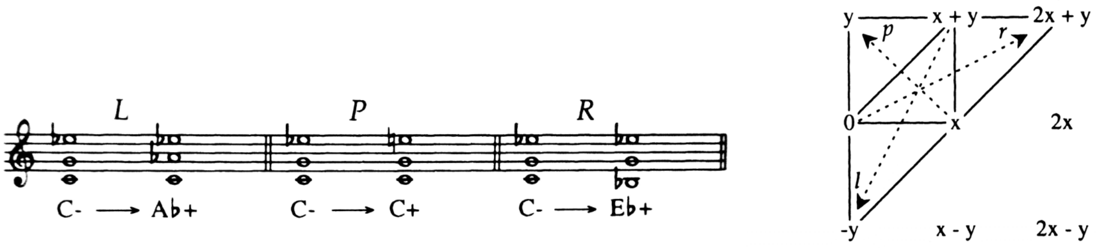

Fig. 11: The PLR-Family of C-Minor (i. e. the smallest harmonic transformations, with a single voice movement) and the corresponding mathematic representation of the PLR-Family operations on a chord Q = {0, x, x+y}.[9]

He made a table showing how the two generative intervals (x,y) where responsible for six main transitional vectors: three in the neighbourhood: x, y and x+y, and three as "voice leading": p: y-x, l: -2y-x, r: 2x+y. According to this table, Richard Cohn find out that the couple of generative intervals (3,4) (i.e. minor and major third) is the only one allowing the PLR relations to describe only "parsimonious" voice movement p=1 (i.e. minor second), l=1 et r=10 (or 2, i.e. major second if octave equivalence is considered).

prime form (0, x, x + y)	step-intervals <x, y, - (x + y)>	*p* y -x	*l* -2y - x	*r* 2x + y
{0,1,2}	<1,1,10>	0	9	3
{0,1,3}	<1,2,9>	$\boxed{1}$	7	4
{0,1,4}	<1,3,8>	$\boxed{2}$	5	5
{0,1,5}	<1,4,7>	3	3	6
{0,1,6}	<1,5,6>	4	$\boxed{1}$	7
{0,2,4}	<2,2,8>	0	6	6
{0,2,5}	<2,3,7>	$\boxed{1}$	4	7
{0,2,6}	<2,4,6>	$\boxed{2}$	$\boxed{2}$	8
{0,2,7}	<2,5,5>	3	0	9
{0,3,6}	<3,3,6>	0	3	9
{0,3,7}	<3,4,5>	$\boxed{1}$	$\boxed{1}$	$\boxed{10}$
{0,4,8}	<4,4,4>	0	0	0

Fig. 12: Determination of the accurate Tonnetz *according to the properties of their relative possible parsimonious relations*[10]

It seems meaningful that a representation of harmonic relations allow at the same time to understand the stretched relations it can have with contrapuntist behaviour. It is

[9] Cohn, Richard, « Neo-Riemannian Operations, Parsimonious Trichords, and their Tonnetz Representations, » Journal of Music Theory 41.1 (1997), 1-66. p. 2 et p. 12. Also : Cohn, Richard, "Introduction to neo-Riemannian Theory : A Survey and a Historical Perspective", *Journal of Music Theory*, Vol. 42, No. 2, (Autumn, 1998) p. 167-180.

[10] *Ibid*. p. 6.

to be noticed that Richard Cohn's table account for cases where intervals are not generative for the chromatic scale (e.g. {0,2,4} or {0,2,6}) and that it seems to avoid some possibilities (like {0,4,9} that leads to <4,5,3>; [1]; [2]; [1]; witch is a solution that has also interesting properties, has we will see further on). To give a more symmetrical aspect to the representation, it is worth applying an affine transformation, a rotation and a symetry, has shown in the following figure.

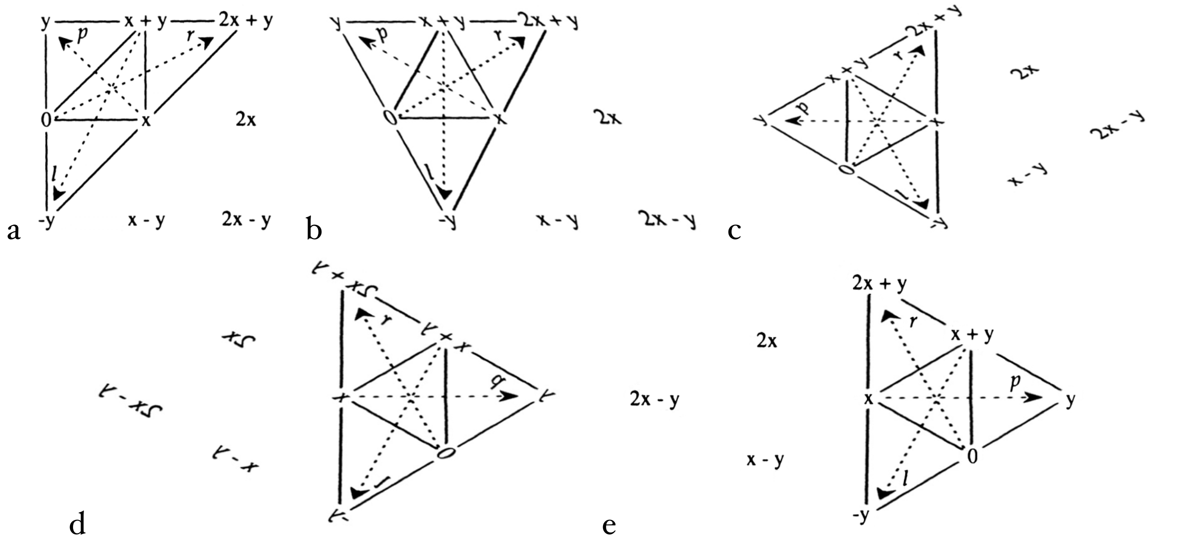

Fig. 13: The theoretical Tonnetz (a) after an affine transformation (b), a rotation (c), and a symetry (d and e).

The last figure (13e) is the one we will refer to from now one. The contribution of Candace Brower, in his paper "Paradoxes of pitch space", leads to a last little transformation, but that is, to our point of view, decisive.[11] Brower make two very interesting statements : he also urges to use a vertical constitution for the cycle of fifths, but proposes to restore the evidence of the chromatic progression by introducing a slight bending of the x axis and thus allowing a meaningful projection on the vertical axis.

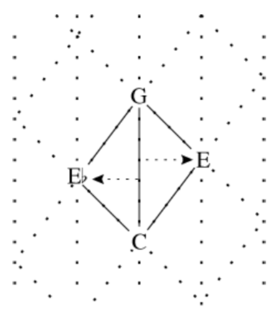

Fig. 14 : The Tonnetz proposed by Candace Brower with the vertical projection of the chromatic progression.[12]

Such a projection was already proposed by James Tenney, but within considerations closer to those of Euler, taking into account a "just" definition of the intervals as

[11] Brower, Candace, "Paradoxes of pitch space", *Music Analysis*, Volume 27, Issue 1, March 2008, p.51 – 106.
[12] *Ibid.* p. 14.

numerical *ratio* and the derivation of musical scales from the numerical definition of the intervals.[13] We will have to comment further one Tenney's ideas.

Even if he is very cautious about the relative properties of visual and acoustic spaces ("we should not expect the visual model to portray directly the embodied qualities of musical objects, but only those features of their geometry that contribute to their image schematic organization"[14]) Candace Brower also proposes an *interpretation* of the *Tonnetz* that is worthwhile to consider. He remarks that "Harmonic and melodic distances tend to be inversely related, so that what sounds melodically close often sounds harmonically distant, and vice versa",[15] and suggest that the tension and relaxation does not work exactly the same at the level of the notes and at the level of chords and that two schemes are possible, one "upward-downward", related to the melodic order, one other, "center-periphery", assuming the "toroid" and cyclic understanding of the chords.

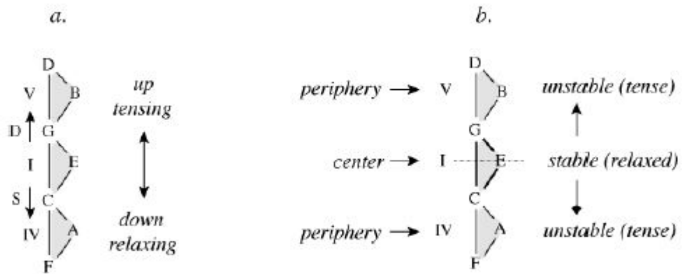

Fig. 15 : Interpretation of the Tonnetz by Candace Brower.[16]

We are induced here to relate this proposal to the diagrams by Hans Mersmann that have been mentioned in the first part of this chapter. But the remarks of Brower are very interesting, has they tend to show that tension and relaxation can *not* be associated with a *single* axis, at least analytically. Is perception making a summary, a subjective projection of all those dimensions on an unique global parameter? Or is it an artefact of our measurement due to our incapacity of acceding, through the language or any other kind of experimentation, to the real complexity of the phenomenon?

Nothing is really clear, in fact, about this theme. The most interesting (and central) example is perhaps the way the "climbing" of the leading tone to the tonic, that should be a gesture of "melodic" tension can be perceived as an "harmonic" resolution, that is to say as a relaxation, even if the harmonic context that could explain this fact is not acoustically present.

Anyway, a chord progression can be visualised on such a representation and the progression can be "projected" both on a vertical axis and an horizontal one. The idea of

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[13] James Tenney, « John Cage et la théorie de l'harmonie » (John Cage and the Theory of Harmony), *John Cage*, revue d'esthétique, n° 13,14,15, Toulouse, Privat, 1987-1988.

[14] Brower, Candace, "Paradoxes of pitch space", *op. cit.* p. 15.

[15] *Ibid.* p. 19.

[16] *Ibid.* p. 21.

"harmonic vectors"[17] proposed by Nicolas Meeùs after Schoenberg's *Structural Functions of Harmony*, could be understood in some extend as the transcription of the harmony's progression on the vertical axis and Xavier Hascher's proposed an interesting interpretation of the horizontal axis as a kind of colouristic substitution axis. The historical evolution of tonal music from a "vertical" discourse illustrated by most of first Viennese school to a more colourful use of harmonic relation initiated by Franz Schubert in the beginning of the romantic area has been evidenced by the work of those researchers. I gave an example of a kind of "apotheosis" of the use of the extended capacities of late tonal system with mesures 16 to 27 of Mahler's Thenth synfony's *Adagio*[18] that can show explicitely how a trajectory in the diagram is the signature of a stylistic an historical behaviour.

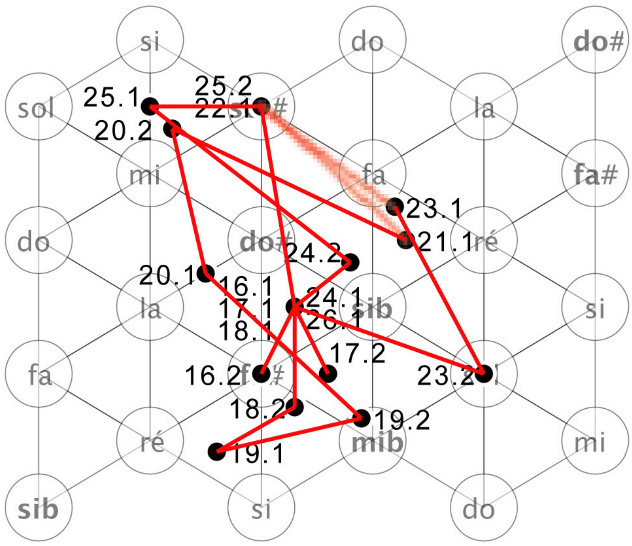

Fig. 16 : Trajectory of the barycenter of the chords used by Gustav Mahler, mesures 16 to 27 of the Adagio *of his Tenth symphony. The point are indexed with the mesure numbers, each mesure being divided in two parts .1 and .2.*

Here, we can see how in a few measures, Malher makes an exploration of all the possible directions around a center (F#). The variety of harmonic solutions, and the « plasticity » of the system is well evidenced by such a diagram.

Even if the choice of the standard *Tonnetz* representation seems to meet traditional requirements for the interpretation of musical harmony, we will now intend to demonstrate that the choice of a representation can be related with a specific musical system. It seems useless to dream of an unique representation that could take into account all the possibilities. In fact the topology of music has a specificity: it deals with several distances. For instance, if you consider that each cycle provides a way to relate

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[17] Nicolas Meeùs, « Toward a post-Schœnbergian Grammar of Tonal and Pre-Tonal Harmonic progressions », MTO (Music Theory on Line, <http://www.mtosmt.org/issues/mto.00.6.1/mto.00.6.1.meeus.html> (27/09/2011) ; Nicolas Meeùs, « Note sur les vecteurs harmoniques », *Musurgia* VIII/3-4 (2001), p. 61-64.

[18] Jean-Marc Chouvel, *Au-delà du système tonal*, actes du Colloque international : « Regards actuels sur la tonalité », Tours, 26-28 november 2009, under publication. See : http://jeanmarc.chouvel.3.free.fr/textes/LeSystemeTonal0.2.pdf

one pitch to another by counting the steps in the cycle, it is easy to understand that one note is always, in some direction, close to the other.

	do	do#	ré	mib	mi	fa	fa#	sol	sol#	la	sib	si
2- (1)	0	1	2	3	4	5	6	-5	-4	-3	-2	-1
5 (7)	0	-5	2	-3	4	-1	6	1	-4	3	-2	5
3+ (4)	0				1				-1			
3- (3)	0			1			2			-1		

Fig. 17: Intervallic distances according to half-tone, fifth, major third and minor third.

The usual {0,3,7} (or, to avoid further confusion, <x,y,x+y> = <3,4,7>) *Tonnetz* allows a good representation of the proximity for the main consonant intervals. It also means that it can help to understand how it is possible to give a single representation of the "pitch" distance (*i.e.* the distance related to the difference of frequencies, clearly represented by the physical distance on the piano keyboard) and of the "harmonic" distance (*i.e.* the proximity of two sounds according to the way they share energy, represented by the concordance[19]). That is what the next figure tends to represent: on its left part, the concordance of an interval formed by a fixed *C* with a varying other frequency, figured by the traditional keyboard. On the right one, a leaning Tonnetz, showing with a black line its underlying continuous constitution. The correspondence in between the intervals for witch concordance is maximum and the ones that surrounds *C* (Do) in the quasi-hexagonal Tonnetz is noticeable.

[19] On the notion of harmonic concordance, cf. Jean-Marc Chouvel, *Esquisses pour une pensée musicale*, L'Harmattan, paris, 1998. It is to be noticed that the calculation of concordance is using a modelised harmonic sound. The aspect of concordance is greatly affected by timbre, harmonicity and spectral thickness. http://jeanmarc.chouvel.3.free.fr/textes/English/HARMONICCONCORDANCE.pdf

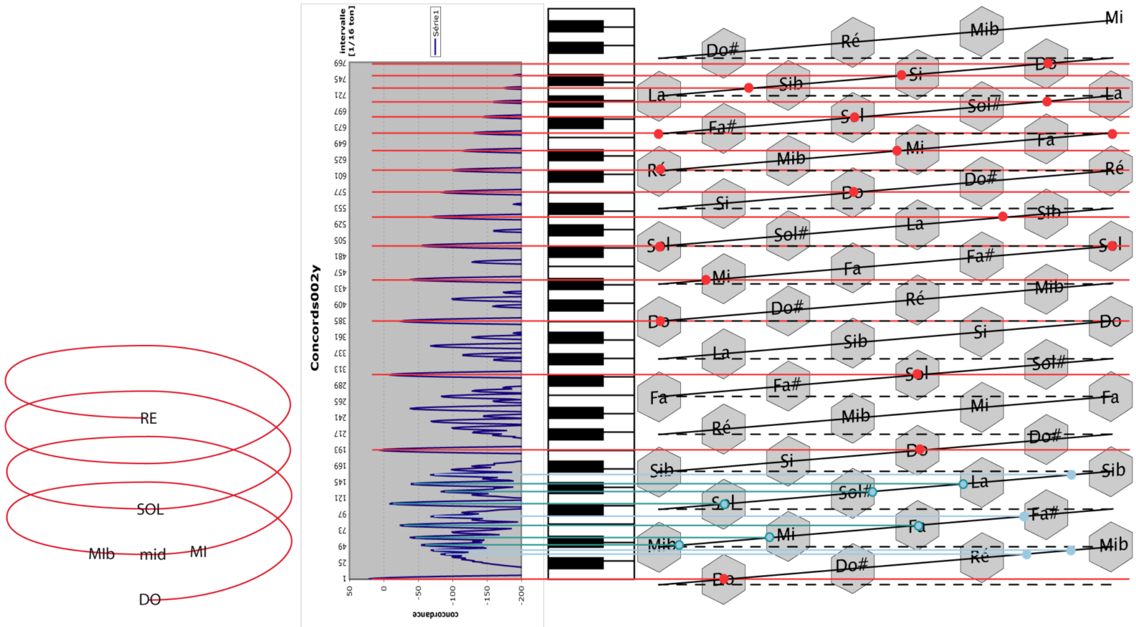


Fig. 18: The usual Tonnetz, illustrated as a continuous spiral equivalent to a keyboard, evidencing the relation in between spatial proximity in the "net", and harmonic proximity (through concordance).

This figure shows that it is possible to understand Candace Brower's interpretation of the Tonnetz as "keyboard equivalent", on a surface that is mathematically a deployed cylinder.[20] The general idea is to introduce a spatial vicinity by "folding" the frequency line so that part of the harmonic relationship in between the frequencies can be honoured by a spatial proximity. Even if the solution presented fig. 17 seems very efficient, it remains impossible to account for *all* concordance possibilities. This means that there are many other ways to construct such a "*netz*". And indeed, music history and instrumental facture shows us that there is not an unique solution to this question.[21] We can have a brief summary of all those intends in the following table:

cylindric coincidence	remarks	**Basic cell**	**theoretical utilisation**	**instrumental utilisation (exemples)**
octave	Square	C_3 $C\#_3$ D_3 ... C_2 $C\#_2$ D_2 ... C_1 $C\#_1$ D_1 ...	concept of "note" equivalence class through octave	superposition of keyboards for the organ
fifth	Square	G_1 $G\#_1$ ... C_1 $C\#_1$ ...	circle of fifths	*Scordatura* for Violoncello, Viola, Violin...
diminished fifth	Hexagonal half octave	C_2 () D_2 ... F_1 () G_1 ()... C_1 () D_1 ...	Kaspar Wicki (1896)[22]	Concertina, Array mbira[23] (with major second scales)

[20] If you consider the notes not as pitches (frequencies) but as symbolic denomination for an octave independant equivalence class, the surface will be that of a torus. See: http://www.musimediane.com/spip.php?article21

[21] See also the review by Steven Maupin, David Gerhard and Brett Park, "Isomorphic Tessellations for musical Keyboard", Proceedings of the SMC 2011 – 8th Sound and Music Computing conference, 06-09 July 2011, Padova, Italy.

[22] http://www.concertina.com/gaskins/wicki/

[23] http://www.thearraymbira.com/arraysystem.php

fourth	Square	F_I $F\#_I$ G_I ... C_I $C\#_I$ D_I ...		*Scordatura* for Bass, gamba (partially), guitar (partially)...
major third	Square	E_I F_I $F\#_I$ ... C_I $C\#_I$ D_I ...	Euler?...	
"neutral" third	Hexagonal half fifth	G_I $G\#_I$ ... Eb_I E_I F_I ... C_I $C\#_I$ ...	Riemann, Paul Dysart,[24] "hexagonal" Tonnetz	AXiS-49, Opal Chameleon or Gecko[25]...
minor third	Square	Eb_I E_I F_I ... C_I $C\#_I$ D_I ...		
5/4 tone	Hexagonal half fourth	F_I $F\#_I$ ... D_I Eb_I E_I ... C_I $C\#_I$ ...		keyboard and chromatic basses of a bayan (accordion)
major second	Square	D_I Eb_I ... C_I $C\#_I$ ...		Harpejji[26]
¾ tone	Hexagonal half major third	E_I F_I $F\#_I$ ... C_I $C\#_I$ D_I ... C_I $C\#_I$ D_I ...		
minor second	Square	$C\#_I$ D_I Eb_I ... C_I $C\#_I$ D_I ...	Paul von Jankó[27] layout (slightly bended)	piano (played inside the instrument)[28]
Unison	"Hexagonal"	() $C\#_I$ () ... C_I () D_I ...	Paul von Jankó layout	piano (locally)

Fig. 19: Table of the possibilities for the construction of a regular net and its use has practical musical arrangement, for "keyboards" or other instruments.

It is amazing to see that we presented as an "optimal" representation is far from being unique. Most of the other possibilities have been used in the course of music history, and not only for theoretical reasons. All the instruments listed here have their own mental, and digital universe, and thus their musical characteristics, their specific way of converting gesture into melody and chords. It is very important to note that the spatial configuration of a *scordatura* implies a specific kind of instrumental possibilities and hence a specific musical "behaviour".

If it is possible to construct other Tonnetzs, that are "keyboard equivalent", it might be interesting to understand better what kind of relations they focuses on and what kind of musical universe they can help to understand better. Within the possibility of constructing Tonnetz, the constraint of being "keyboard equivalent" selects the steps intervals (we prefer here a reference like <x,y,(x+y)> slightly different with the one chosen

[24] http://tonnetz.zxq.net/index.html
[25] http://www.theshapeofmusic.com/
[26] http://www.marcodi.com/
[27] Paul von Jankó, "Neuerung an der unter no25282 patentirten klaviatur", German patent office, Tech. Rep. 1885.
[28] The standard keyboard is not regular, but is not so different, locally from the Jankó layout.

by Richard Cohn: <x,y,-(x+y)>) in such a way that y-x=1, i.e. that the difference in between steps intervals is one unit of the scale. It means that the interval defining the cylindrical progression of the representation is (2x+1)/2 like, or in other words, that the reference interval (x+y) is a “odd” interval, like minor third (3), fourth (5), fifth (7) (the usual Tonnetz representation), major sixth (9), major seventh (11), minor ninth (13), and so on…

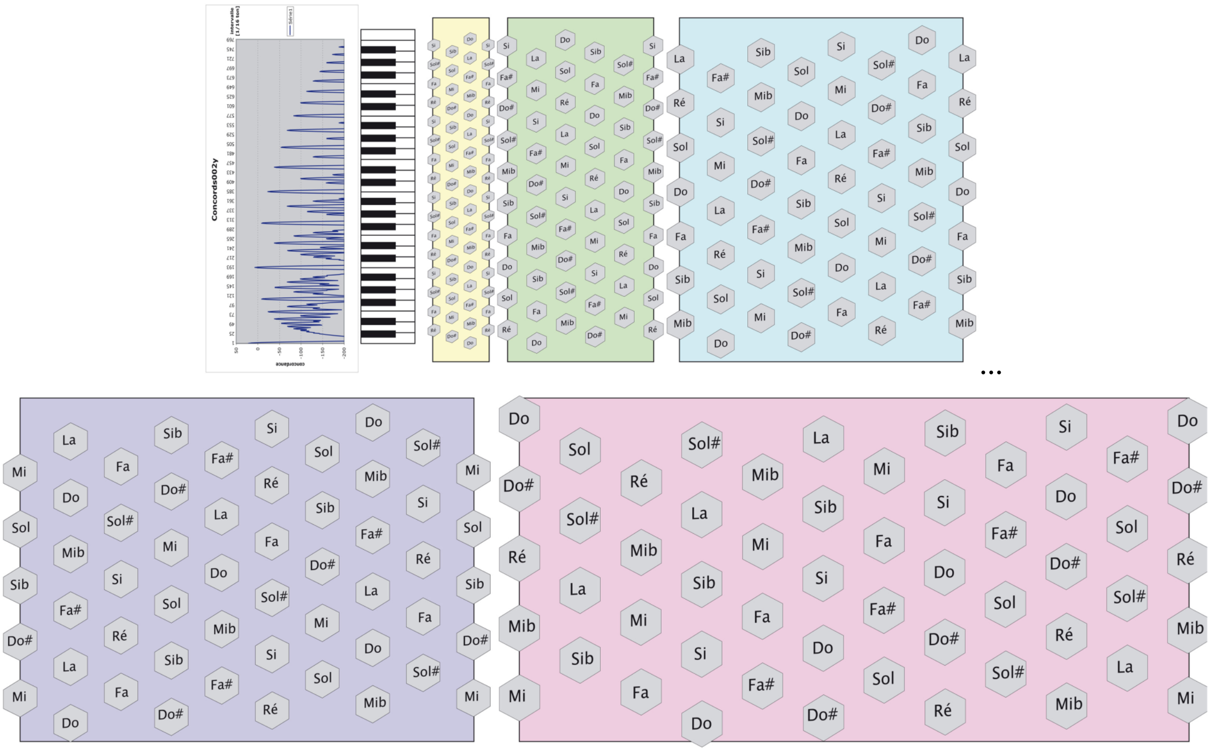


Fig. 20: “Keyboard equivalent” Tonnetzs, for <x,y,x+y> = <1,2,3>; <2,3,5>; <3,4,7>; <4,5,9>; <5,6,11>.

Those representations leads to reconsider the concept of “major” and “minor” chords. For each representation, there is a specific couple of compact triangular figures, that varies according to the steps intervals values. It is interesting to re-consider what is supposed to be the fundamental basis of our concept of harmony:

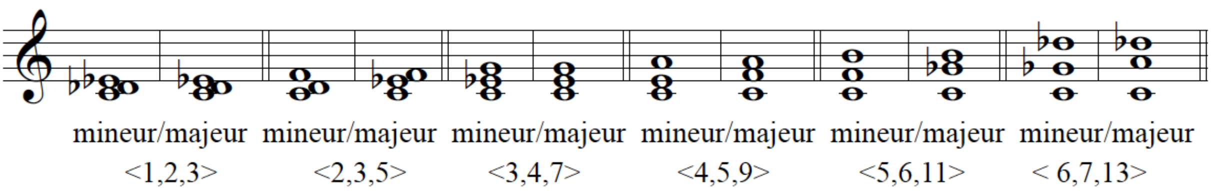


Fig. 21: “major” and “minor” chords for “Keyboard equivalent” Tonnetzs, <x,y,x+y> = <1,2,3>; <2,3,5>; <3,4,7>; <4,5,9>; <5,6,11>; <6,7,13>.

If we represent in white the diatonic scale, and in black the complementary pentatonic scale, like for the keyboard of a piano, we can have a very interesting overview of the way each one of those representations can focus on certain kind of relations, and imagine that it can provide interesting developments on the possibility to have several harmonic musical systems with the same degree of coherence than Tonality… <3,4,7> and

<4,5,9> are different as “keyboard representation” – “major” and “minor” chords of <4,5,9> corresponding to the “first inversion” of the “major” and “minor” chords of <3,4,7> – but remain equivalent as “toroid” harmonic representation, through a rotation.

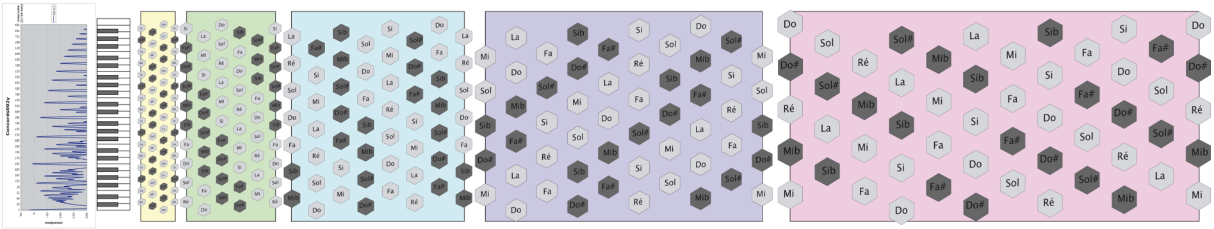


Fig. 22: “Keyboard equivalent” Tonnetzs, for <x,y,x+y> = <1,2,3>; <2,3,5>; <3,4,7>; <4,5,9>; <5,6,11> evidencing the diatonic and pentatonic scales.

It is very interesting to compare the second representation <2,3,5> to the third <3,4,7>, has they seem to draw similar vertical “columns”, specially if the dark column of <2,3,5> is compared to the white one in <3,4,7>. This can mean that the <2,3,5> set is likely to describe the pentatonic scale in the same way that the <3,4,7> is meaningful for the diatonic system. But there is one main difference: on the <3,4,7> set, two parallel vertical columns can be evidenced, while there is only one on the <2,3,5>. What does this mean? In the case of diatonism, it means that the chords are made with one note of two in the scale leading to distinguish a cycle (of thirds) for harmonisation different of the cycle (of seconds) of the scale itself… If we conserve the same idea, we are lead to consider that there is not such a distinction with a pentatonic scale and therefore that it can be harmonised with the consecutive steps of the scale, a statement that any little child playing on the dark keys of a piano has experienced. It is a good example of the way Tonnetzs reflects important structural properties of the musical systems they can be related to.

It is not worth it, here, to enter in a complete description of how the tonal system can be reflected in the standard neoriemanian Tonnetz, but it seems more useful to give two very small examples illustrating the potentiality and the signification of the other Tonnetzs. The first example is taken from the very beginning of Liszt’s famous piano work entitled *Nuages gris* or “*Grey Clouds*”, a work he wrote at the age of 70 that is often considered as a foreshadow of Schœnberg’s music for its taste of atonality.

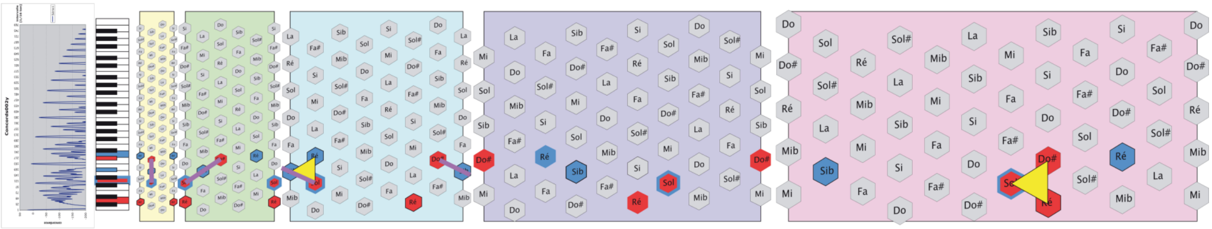


Fig. 23: The beginning of Liszt's Grey Clouds *and it's representation (the first three notes are red and the last ones are blue, yellow triangle are for triads).*

It is interesting here to see that each fragment of this line gives a triadic allusion corresponding to a different harmonic system, but in each case intended to be "minor" triads. Even if a traditional "tonal" analysis of this theme remains perfectly possible, it is worth saying that it also deals with something else… What seems interesting here is to imagine that all those netzs are specific spaces where some harmonic (i.e. relational) properties can be projected.

The second example is taken from Edgar Varèse, a small part of the second instrumental episode of *Deserts*. Chords have been represented in a <6,7,13> Tonnetz. This Tonnetz is very different from the "standard" Tonnetz, but it offers a very consistent represention of the specific harmonic construction (and language…) chosen by Varèse, where disminished fifth, fifth and minor ninth have nearly the same function as thirds and fifth for tonal music…

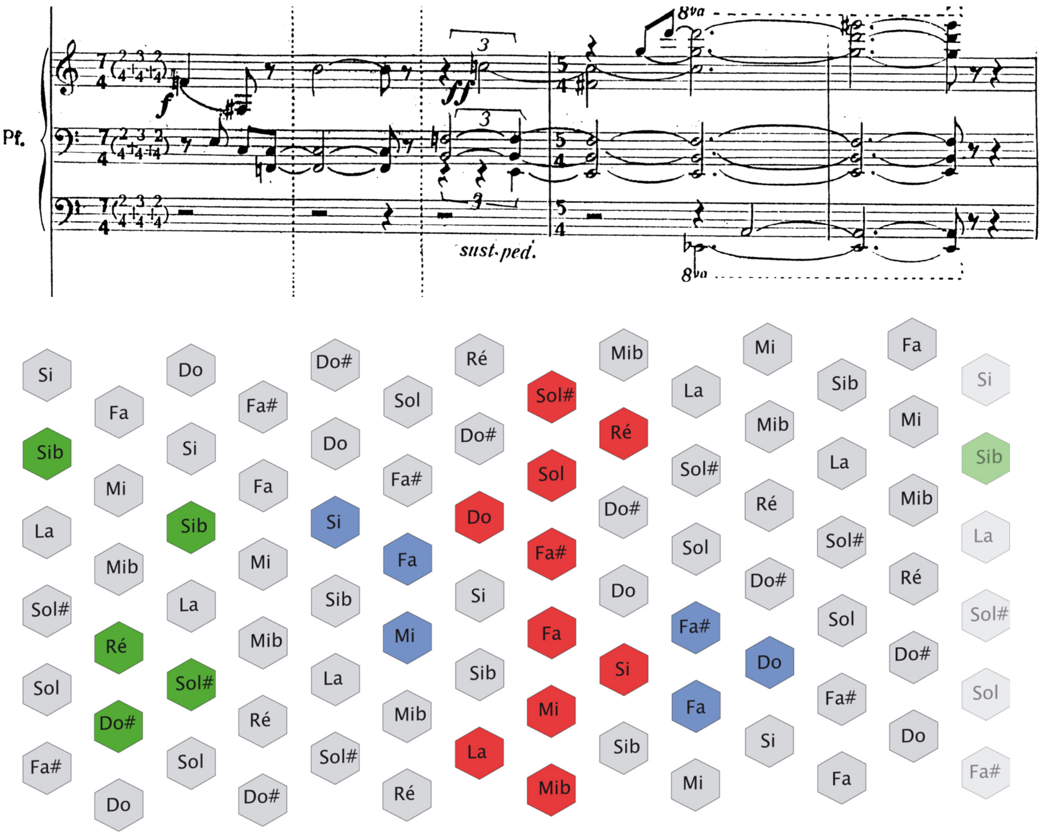
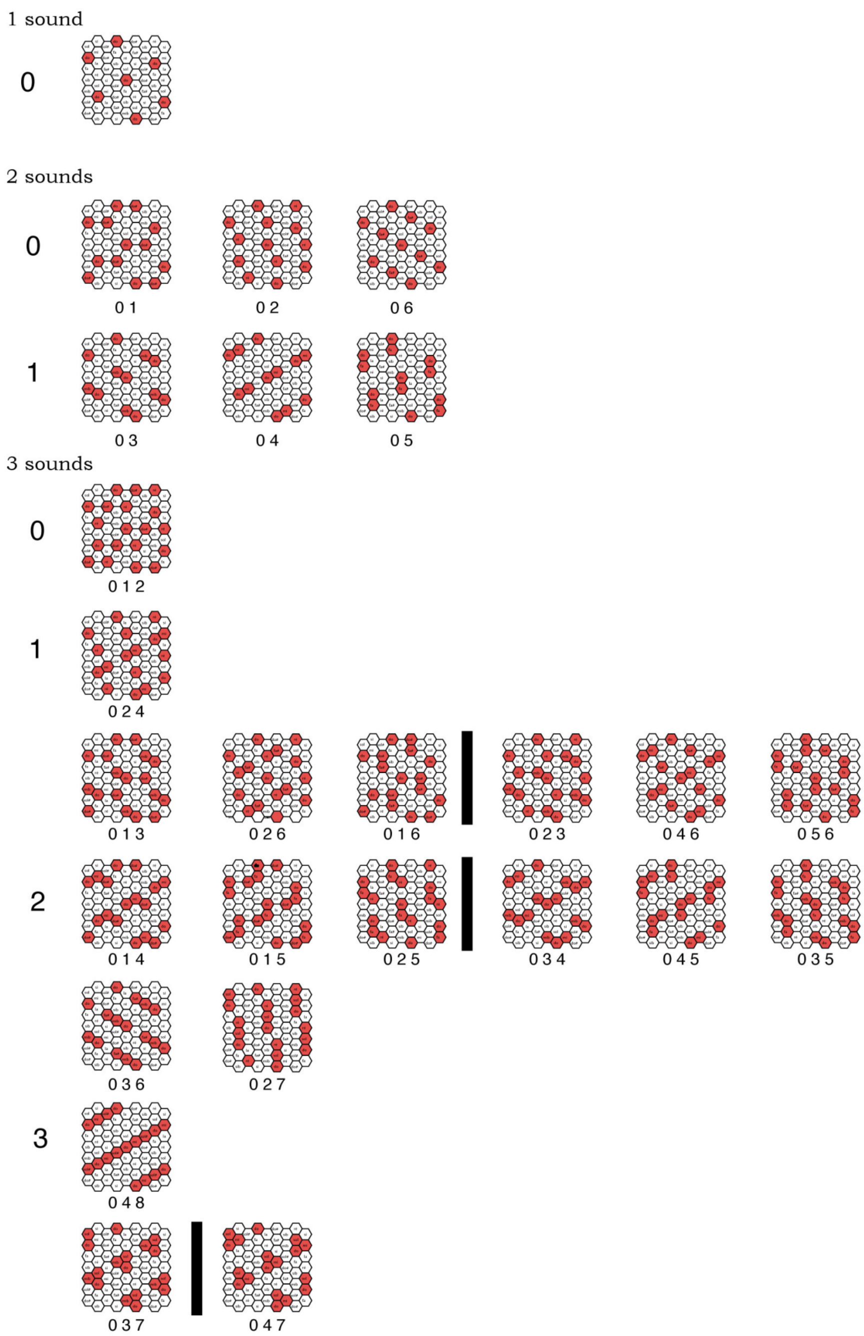


Fig. 24: Déserts *from French composer Edgar Varèse, bars 115-117 and the corresponding chords in a <6,7,13> Tonnetz. Green: former harmony; blue: bar 115.1&2; red: bar 115.3 to 117.*

3. Topology and structure.

The former representations where concerning the level of musical sound called notes. The superposition of those notes was drawing a figure that was the representation of a chord. Some chords are privileged within a specific *Tonnetz* forming a family of traditionally named "perfect" chords, has they are involved in the basic structural constitution of the *Tonnetz*. There are two kind of "perfect" chords, "minor" and "major", according to the vertical symmetry and the interval order. But there is quite an important number of chords that does not fit to those basic grouping. In the <3,4,7> *Tonnetz*, the representation of those chords can lead to the following enumeration, according to the number of notes, and as a second criterion, to the number of connexion to a neighbour:



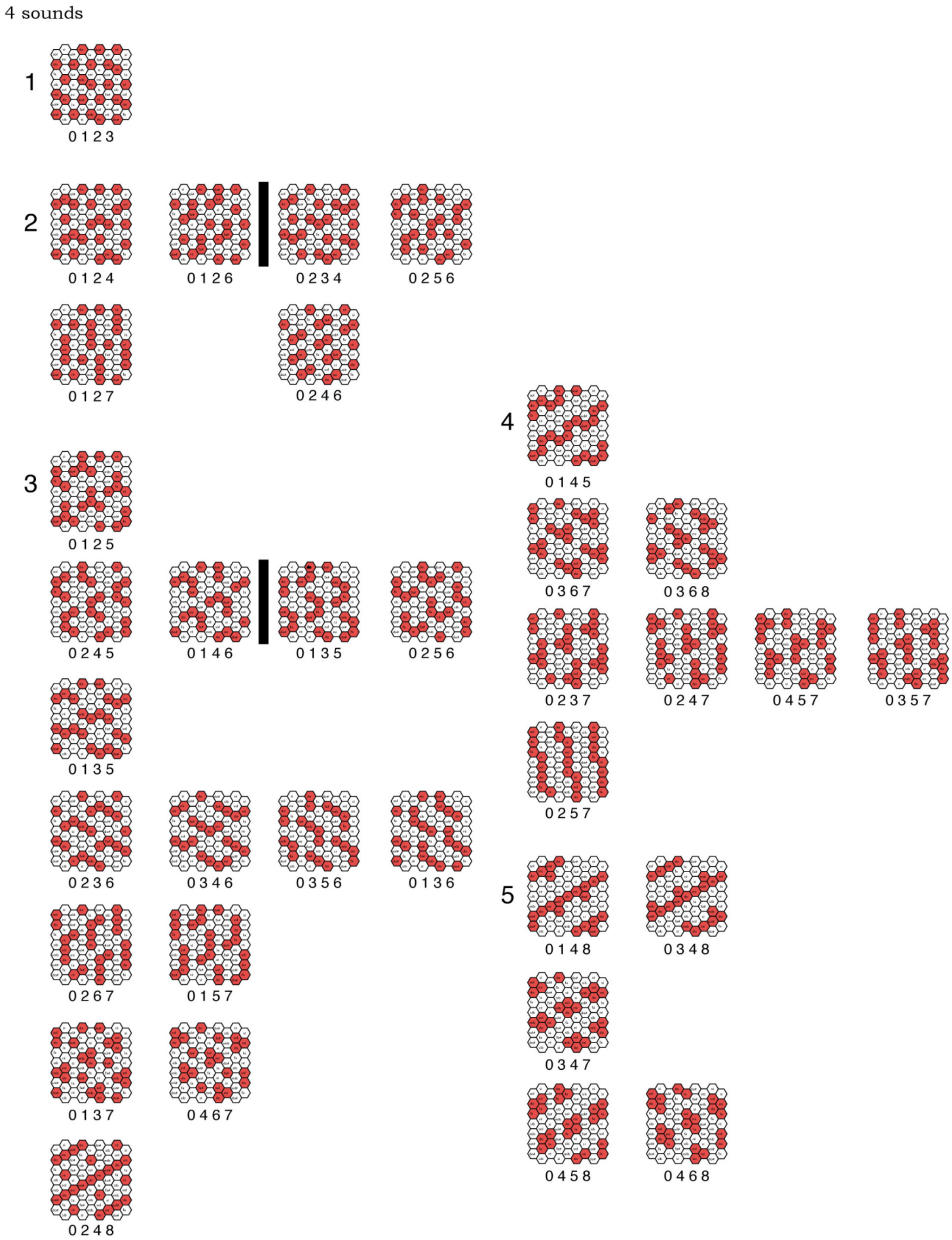


Fig. 25: Representation of the chords on a <3,4,7> Tonnetz (toroidal mode). The lateral numbers classify the chords according to the number of proximity "connexions" through the net and the numbers under the figure give the composition of the chords (constitutive intervals)

This nomenclature has the benefit to be visually easier to identify, and, of course, transposition independent. It also provides a hierarchy of the notes cohesion within the net, witch should be harmonically meaningful, according to the hypothesis of a relation in between neighbourhood on the net and concordance. And other way of seeing it is to come back to the kind of presentation shown fig.7. This allows a better understanding of the global symmetries within the different chords in the *Tonnetz*.

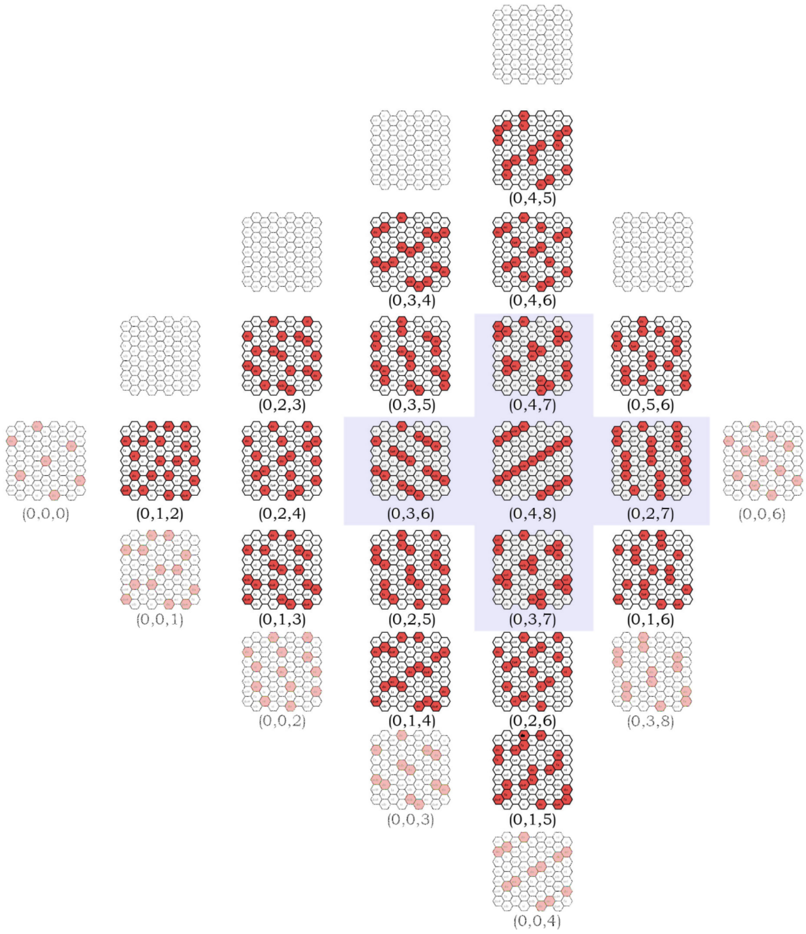


Fig. 26: Representation of the three tones chords on a <3,4,7> Tonnetz (toroidal mode) with evidence of the possible symmetries. The shadowed solution are not real three tone chords as two (or three) notes are similar.

The central underlined chords in this setting are the ones that correspond to the logic of a given *Tonnetz*. Those chords can be named “triads”, and they are either the compact “major” and “minor” chords, or the chords specified by the redundancy of the three main “interval directions” of the *Tonnetz*, that is to say, in the case of the <3,4,7> *Tonnetz*, superposition of fifth, major third and minor third. This leads to an other interpretation of the *Tonnetz*, where what is represented is not only the tone itself, but this specific set of tone we called “triads”. Any kind of chord, specially complex chords that have more than three tones, can be (or not) decomposed with those specific sets. Perceptually, each triad has a very specific “colour”, quite easy to distinguish, and the melting of those colours is understandable has the superposition of each of them. This can be a way of having a structural understanding of harmony.

Fig. 27: Representation of the triads for "Keyboard equivalent" Tonnetz.

This is possible of course for every *Tonnetz*, but it has a specific historical relevance for the <3,4,7> *Tonnetz*, witch can be associated with what is called "tonality". We can briefly give here some clues explaining this specific relation, but it seems important, before that, to come back to the relation in between concordance and *Tonnetz*, at this specific level of the triads. It is possible to visualise the concordance of three notes (for a specific definition of the spectral behaviour of those notes) as a kind of map giving as a "altitude" axis the concordance while the plane coordinates are giving the defining intervals of the triad [x=first interval; y=second interval; z(grey levels)=harmonic concordance]. The result is shown on fig. 28, using the same conventions used in fig. 27.

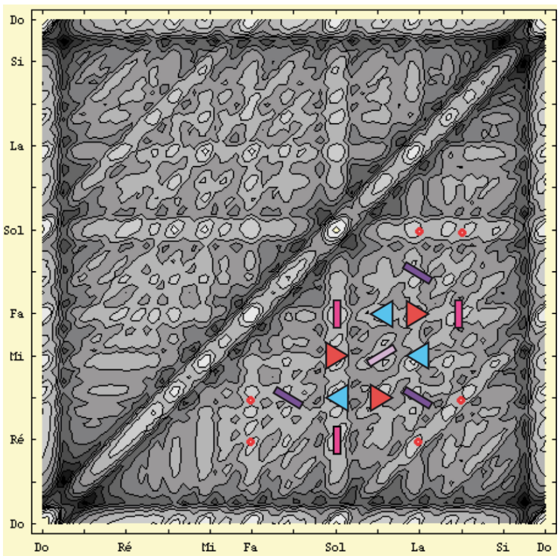
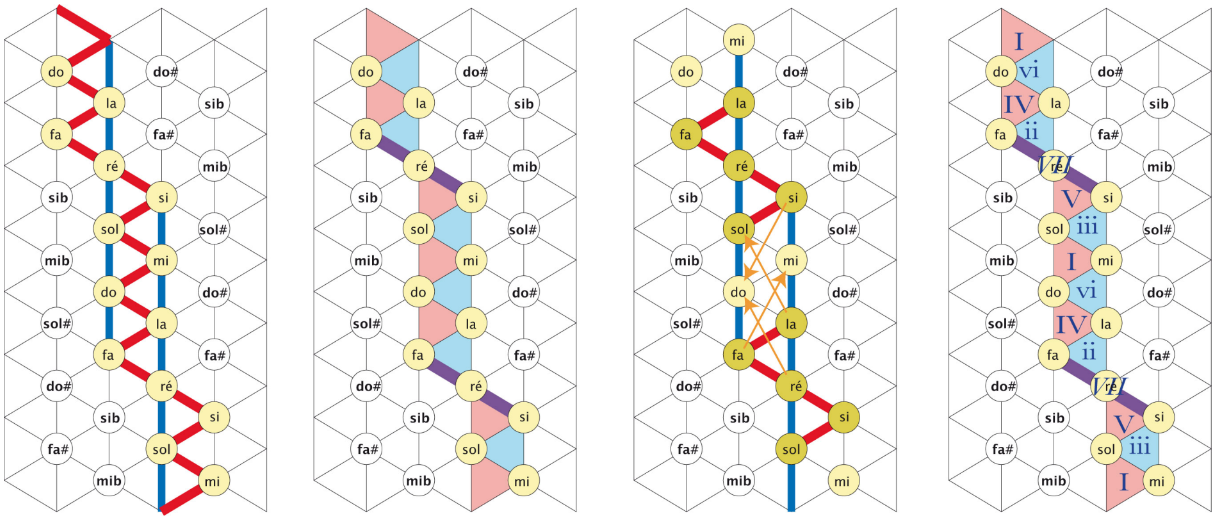


Fig. 28: Triads and concordance. The plot is showing concordance of three notes : (C, C+x, C+y) and the positions of the triads for a <3,4,7> Tonnetz*, using the conventions of fig. 27. The round positions correspond to a defective form of fifth cycle.*

It is very clear on this representation that the triads of the <3,4,7> *Tonnetz* have a great symmetrical positioning on the concordance diagram and that they are fitting with the main concordance maximum, even if some approximation is required.

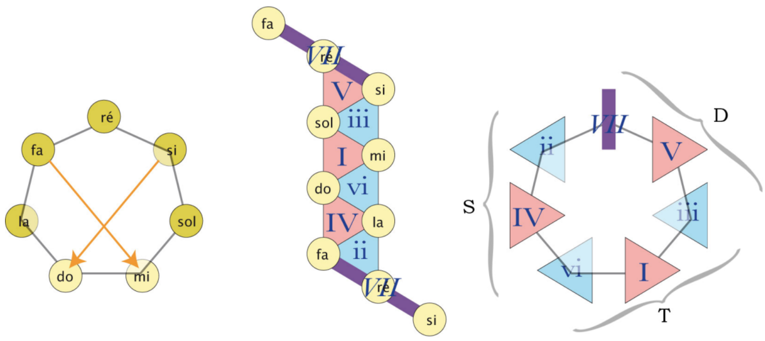
We can now have a look on the third level of structure, that of tonality itself. The diatonic system that gives tonality its own “shape” is represented on the <3,4,7> *Tonnetz* by an asymmetrical ribbon centred on D. Some theorists (e.g. see fig. 7) assimilate it to a Moebius strip. The third and fifth cycles that generate the diatonicism are clearly appearing on the representation (fig. 29(a)) as well as the constitutive triads (fig. 29(b)), that can be assimilated with the “degrees” in French ramist type notation (fig. 29(d)). The central paper of V to I cadence is quite easy to understand, as well as the “parsimonious” melodic lines that underline this fundamental progression (fig. 29(c)).



(a) (b) (c) (d)

Fig. 29: Major C tonality in the <3,4,7> toroid Tonnetz. 29(a): the cycle of fifth (vertical) and the cycle of thirds (diagonal); 29(b): the triads within diatonism; 29(c): the main melodic relations characterizing Major C tonality; 29(d): triads as degrees.

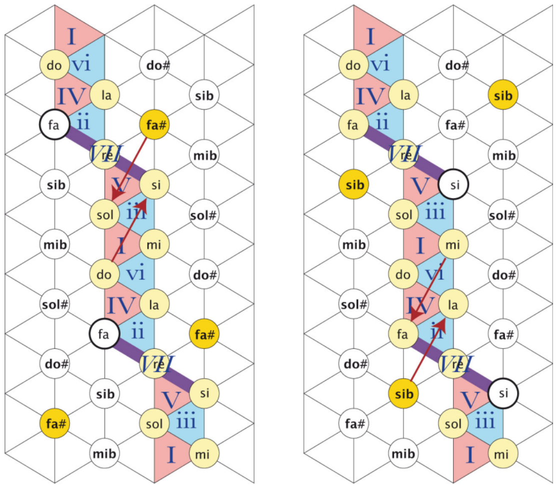
The functional relations are also well evidenced, specialy if the toroid nature of the representation is correctly understood. Other “sights” of the representation can be seen on fig. 30., especially the Hascher’s cycle (fig. 30c) evidencing Riemann’s functions.



(a) (b) (c)

Fig. 30: Diatonism as cycle of thirds.

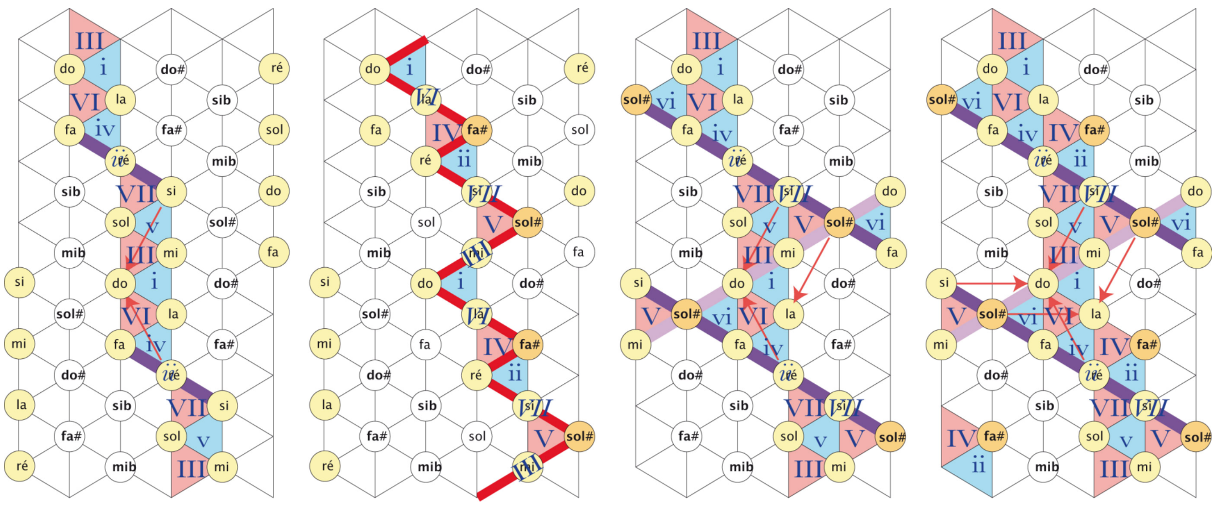
It is also very easy to understand the different modulations in between tonalities.



C Major towards G Major — C Major towards F Major

Fig. 31: Adjacent modulations in C major.

But if everything seems quite obvious about major tonalities, it becomes far more complex if we try to understand how this works for minor tonality. There is no apparent difference for the ascending scale, that is conform to standard diatonism, except that the degrees are numbered differently (fig. 32(a)). But with the use of the descending scale, the cycle of thirds takes a quite different shape, and allows a great number of specific triads (fig. 32(b)). Then the degrees and melodic possibilities are far more important for minor tonalities than for major ones, as fig. 32(c) and 32(d) can evidence. It is worth noting that with two more tones, and many more triads (of all kind), minor tonality offers another kind of exploration of tonality, less related to the verticality of major-like progression.



(a) (b) (c) (d)

Fig. 32: A minor tonality in the <3,4,7> Tonnetz. 32(a): an other name for the degrees conserving the diatonic scale; 32(b): the ascendant scale as thirds cycle and the corresponding other triads generated; 32(c): the triads generated by the "majorisation" of the v degree (G#); 32(d): all the possible triads within the full A minor tonality. Most degrees have at least two forms.

4. Representation of microtonal worlds.

The aim of a representation is not only to provide an explanation to what is already well known, but also to allow an exploration of other possibilities and other systems. This prospective part is in fact essential to understand the historical motivations of a theory, but it is often relegated to a pure speculative and even subjective way of seeing. The use of the representations related to *Tonnetz* for Musical Analysis remains to explore, even if there are already very interesting examples. But it is always important to know whether a theory applying to some domain can be generalized and also complies with others. This makes usually the strength of a scientific proposition, and scientists often look for a more general vision that can assume the description of the known world but also account for the part of the world that is not already familiar. It is mostly the purpose of modelling and representation: a way to anticipate under a manipulable scheme inaccessible data.

The neo-riemanian constructions where designed in the context of a discrete mathematical symbolic representation of music. But the reality of musical sound is continuous, and then a theory restricted to "well-tempered" occidental scale, even within the scope of global standardisation of human live, will always be partial and limited. This theory has been presented here in such a way that it was clear that under its discrete configuration was laying a continuous universe, and that the chromatic line was a dotted line on the reality of music. Now remains the following question: how can the *Tonnetz* representation be extended to microtones?

It is not possible to attend here all the different historical propositions for microtonal musical systems.[29] We will give three directions, demonstrating the diversity of approach a powerful theoretical tool is capable to synthesise. First we will show that the traditional definition of intervals as *ratio*, deriving from considerations on harmonics, can be interpreted as a red, and is then legible on the "spiral" *Tonnetz*. The second aspect we will focus on will be the structure of Arabic scales. Then we will try to imagine what *Tonnetz* can be for micro-intervallic world like sixth tone scales. A lot has to be done to understand the harmonic behaviour of those musical spaces.

The fact that the *Tonnetz* where first proposed by Euler as a representation of the difficulty of working with just intonation and looking for an equal temperament is probably the best way to introduce the importance of this problem and its relation to the ideas of "*netz*". If the frequencies of an harmonic spectrum are superimposed to a "keyboard equivalent" *Tonnetz*, it is very easy to see the way the intersections with the chromatic line does not fit with the regular "tempered" grid. Each one of the intersection point can define a vector, and the "just intonation" system can be understood as the set of frequencies defined by the combinations of those vectors. A set of harmonics (or a set

[29] An extensive bibliography can be found here: http://www.huygens-fokker.org/docs/bibliography.html

of odd numbers) defines a subset of frequencies that leads to a regular plot on the *Tonnetz* that differs from the well-tempered one. This kind of representations had been proposed by James Tenney, in his precursor work on harmony,[30] and could be used for the realisation of just intonation percussive instruments.

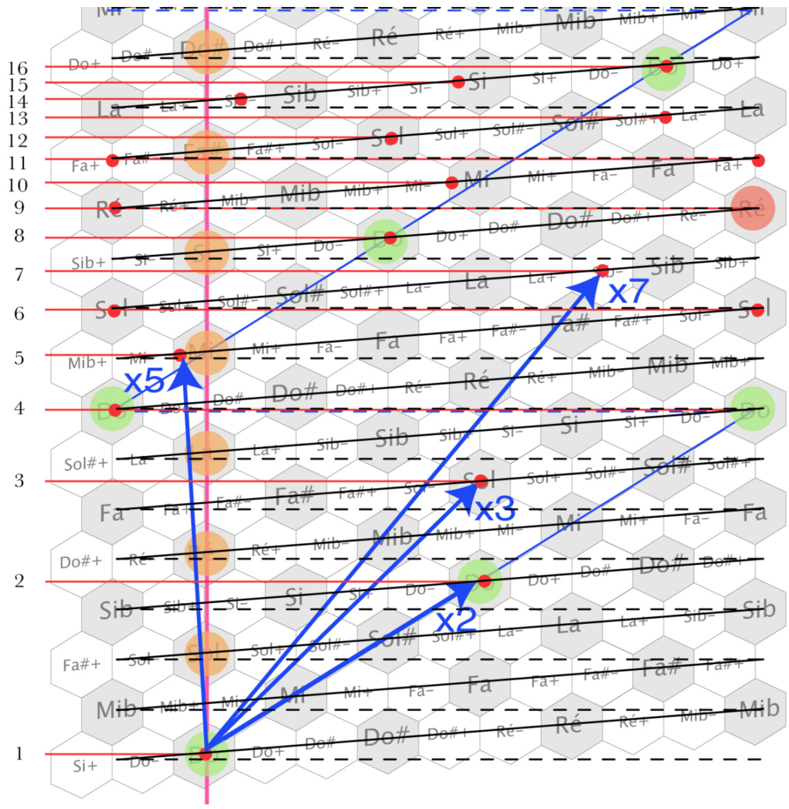


Fig. 33: Representation of ratio *intervals within the extended* Tonnetz. *Any interval defined as a* ratio *can be represented as the combination of the corresponding vectors. This can help to understand the constitution of just-intonation universes.*

Non-tempered scales can be represented on *Tonnetz* with slight differences of position and an evaluation of their harmonic properties can be provided by the use of concordance tables.

Another interesting generalisation of *Tonnetz* concerns Arabic scales.[31] We will not speak here of the subtle non-tempered feeling of old Arabic *maqam*, knowing that they can be approximated exactly the way we happened to describe. Since the twentieth century, this music has been theorized as scales in a tempered system, and its structure described as the concatenation of fragments called jins or *ajnas*.[32] Being aware that what follows is not the traditional description of a traditional music, let's have a look of the behaviour of the different *maqams* in the expended theoretical frame of the *Tonnetz*.

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[30] James Tenney, « John Cage et la théorie de l'harmonie » (John Cage and the Theory of Harmony), *op. cit.*

[31] The following considerations are deduced from the work of the Syrian composer and musicologist Waël al-Nabulsy, *La catégorisation des échelles dans la musique arabe du Machreq au XXe siècle : approche théorique et épistémologique*, Université de Paris-Sorbonne, may 2010.

[32] For such a description see : http://www.maqamworld.com/ajnas.html

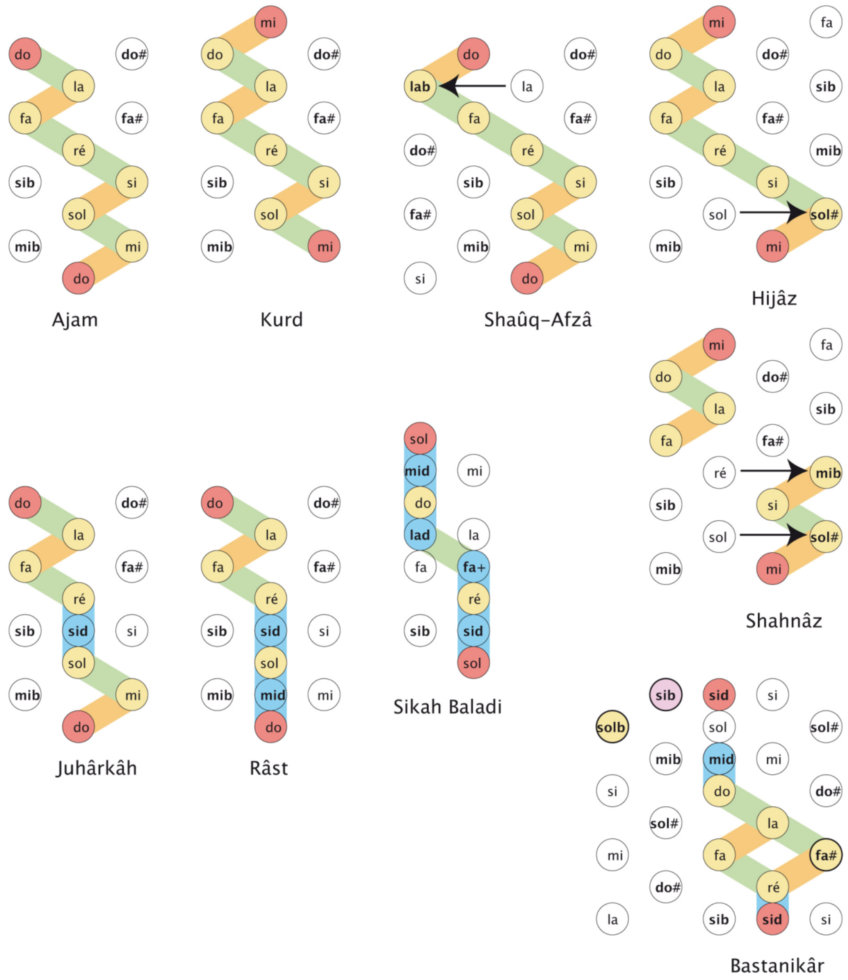


Fig. 34: Representation of some Arabic Maqams *in the <3,4,7>* Tonnetz.

The *Ajam* scale is a diatonic scale similar to C major. It is represented with a third cycle that has already been commented. The diatonic scale can begin on another degree, like the *Kurd* scale, beginning on E, and thus equivalent to the modes we call the *phrygian* mode or E mode used by western church. All those diatonic modes are coherent with the fifth cycle and then can be transposed using the common alteration system: $B \to Bb$, $E \to Eb$, ... ; $F \to F\#$, $C \to C\#$, ... They are presented here according to their minimal alteration form. For example, the *Kurd* mode is usually presented in D, that is to say with two flat tones and is here transposed to E, where it has no alteration. All the other modes we will explore now are not diatonic, but we will still refer them to a diatonic structure, has it has this useful property for transposition. The scale called *Shaûq-Afzâ* can be understood has a deformation of the *Ajam* scale, with the adjunction of a non-diatonic alteration to the sixth degree $A \to Ab$, that leads to an emphasis of the minor third cycle in the whole scale. Another possibility is given by an ascending the third degree of a *Kurd* scale: $G \to G\#$. This makes an *Hijâz* scale on E. In fact those two solutions leads to the same intervallic structure, has it is well evidenced on the *Tonnetz*, that respects a continuity of the thirds cycle. Whenever an other alteration is used, this cycle is interrupted. This is the case of the *Shahnâz* scale, witch has a second non diatonic alteration: $D \rightarrow D\#$ added to the former $G \rightarrow G\#$. This mode presents more major thirds than minor ones, and thus, introduces a major second in the cycle. It is worth noticing that this scale includes two major seventh chords, separated with a minor third.

All the former scales are coherent with chromatism, and then can be played on a standard piano keyboard. The three scales we will focus on now introduce a quarter tone alteration. Those quarter-tones does not appear in a erratic order and are also related to fifth cycles. The first one is then $B \rightarrow Bd$, d being here a descending quarter-tone alteration. The scale called *Juhârkâh* is corresponding to this first modification. There are no difficulties to represent this on the extended *Tonnetz*. It becomes obvious that the third cycle is respected, with the transformation of an alternation of major and minor thirds into a couple of "neutral" thirds. If two quarter-tones alterations are used, like in the very popular *Râst* mode, the phenomenon is even amplified, but the same logic seems to be continued, till an ultimate form that could be represented by the scale called *Sikah Baladi*, where four quarter-tones are used, respecting the idea of a third cycle even if in this mode, "neutral" thirds are the absolute majority.

The ultimate example that will be commented here is a mode called *Bastanikâr*. This mode is rather unusual because it is not really possible to transform it into a scale, as the octave equivalence is not respected. This mode begins with Bd and ends with Bb. But it is not the only strange property of this mode. It has two regular quarter-tones, like *Râst*, but the irregular transformation $G \rightarrow Gb$ leads both to a rupture of the thirds cycle (like in *Shahnâz*), but also to a major-minor ambiguity in the cycle.

It is not possible here to draw in some lines a complete theory of a very complex music. Nevertheless, it is worth noting that the *Tonnetz* give a brand new way of understanding Arabic *Maqams*. Modal music is thought through melodic relations, and traditional Arabic music is played and learned and analyticaly described this way. But the use of polyphonic instruments, the cross-cultural use of philharmonic orchestra, and then the apparition in the course of the twentiest century of an harmonised arabic music leads to new challenges. It will be interesting in the future to see if the fact of benefiting from a representation is able to help the development of a new way of understanding a musical tradition, where centuries of intuition and genious are usually necessary.

Arabic musicians are not the only ones to be in need of a theory to go ahead with micro-intervallic new musical worlds. Contemporary composers, in the western tradition, even if globalization makes it not so clear whether the word "western" is relevant, are already using very often micro-intervals. There are two main streams of awareness about how to theorize this use. One is giving a priority to the convenience of the tempered world, using its traditional notation adding only some specific alterations. The other is referring to the harmonic spectrum, giving a preference to the "natural" ratio, even if it is often "approximated" with the same traditional notation. In fact both conception are not so incompatible. The harmonic spectrum itself has some properties that are worth noting, and that can be evidenced in the following figure.

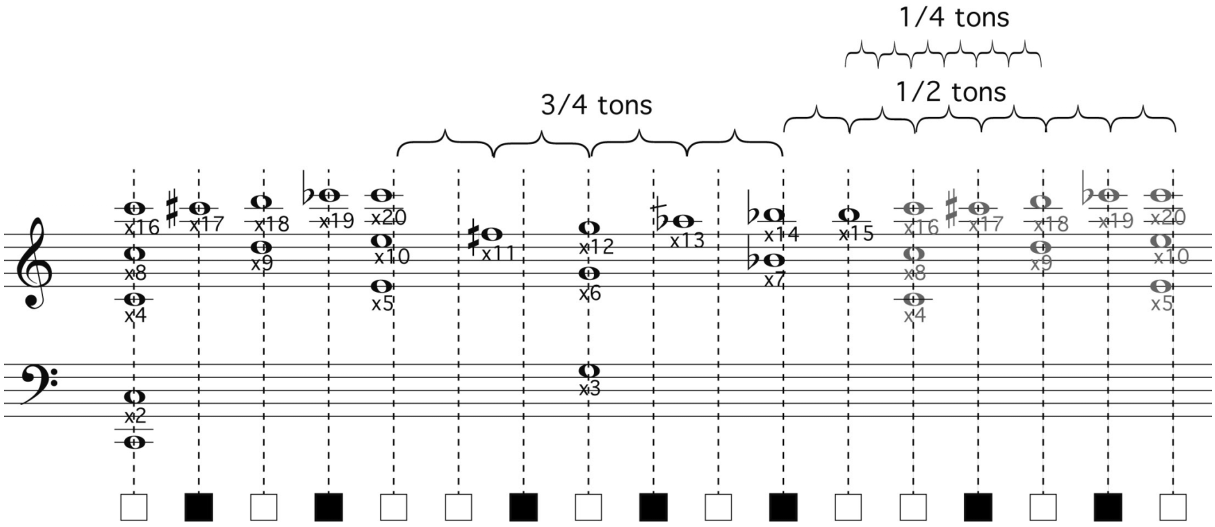
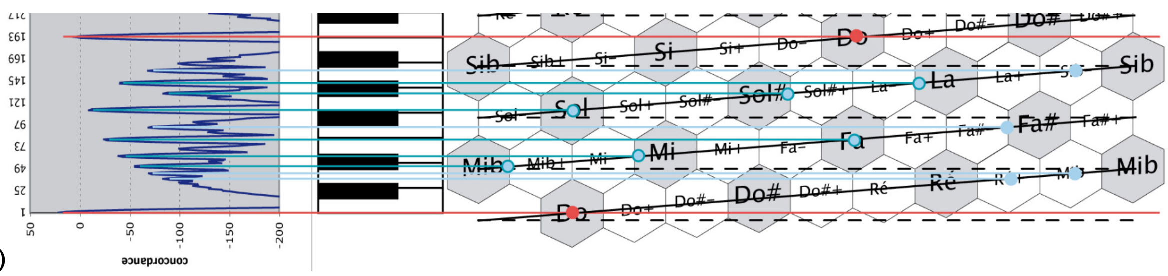


Fig. 35: The harmonic spectrum, according to an octave redundancy.

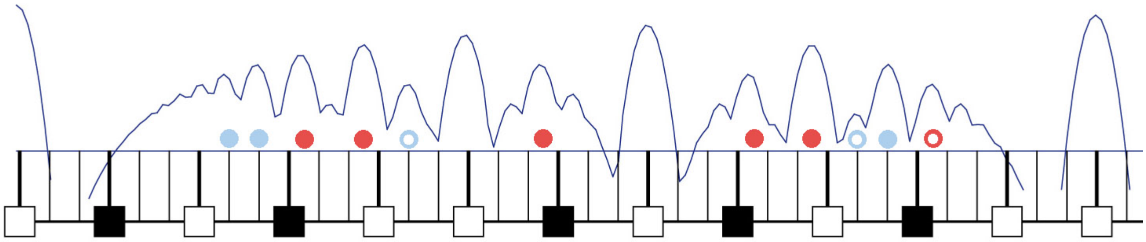
There are two zones where the spectrum can be approximated with a regular scale : around the fifth, with minor thirds and ¾ tones, and around the octave, with major thirds, tone, ½ tone, ¼ tone, etc. This is coherent with the emphasis on ***superparticularis ratio*** $(n+1/n)$ that are in fact nothing else than the interval between two consecutive partials in the spectrum. It can also explain the way Arabic music uses quarter tones, mainly by a subdivision of minor thirds.

But this way of seeing if giving a great importance to very high partials, that in fact do not account for such a great amount of energy in the signal. It would be far more realistic to come back to the concordance itself on this subject. When we have a look to the way the concordance maximums are corresponding to specific frequencies (cf. fig. 18), it is not obvious at all that quarter tones have something to do with it, has no concordance maximum does correspond to a quarter tone at all… Sixth tones seem a much better solution for a scale harmonically coherent under the chromatism. Some intervals (C&D+; C&Eb-; C&Bb-)[33] seems very correctly approximated by sixth tones, and others (C&Eb+; C&E-; C&F#-; C&G#+; C&A-) are not more approximated than the regular half tone intervals (C&Eb; C&E; C&F#; C&G#; C&A).



(a)

[33] X- is a note minus 1/6 tone ; X+ a note plus 1/6 tone.



(b)

Fig. 36: (a): extract of Fig. 18: The usual Tonnetz, *evidencing the relation in between spatial proximity in the "net", and harmonic proximity (through concordance); (b): Correspondence between concordance of two notes and a sixth tones scale.*

It is possible to built an hexagonal *Tonnetz* for sixth tones, exactly the same way it was possible for quarter tones with a squared net, respecting the chromatic construction. The problem is then how to give it a structure, hence the number of chords of Three sounds is very important, and the number of possible figures on the *Tonnetz* less easy to circumscribe. It is not really possible either to expect a kind of similarity in between the geometric distance on the sixth tones *Tonnetz* and concordance...

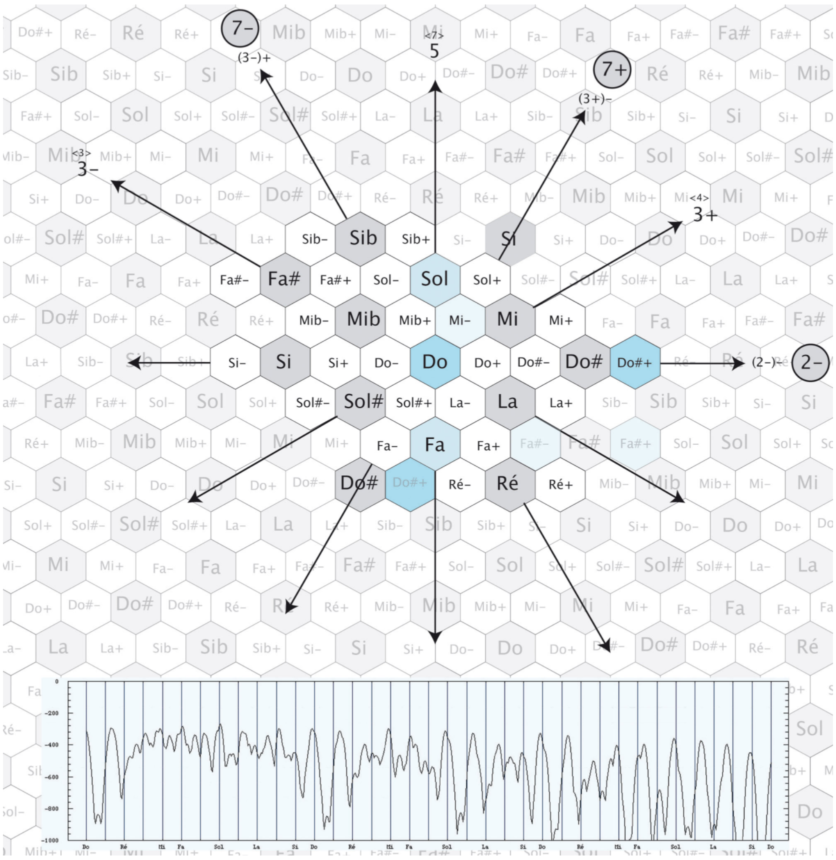


Fig. 37: The <3,4,7> Tonnetz extended to sixth tones, showing (blue levels) the concordance of a chord including C, C#+ and a third note (on x axis in the lower concordance diagram).

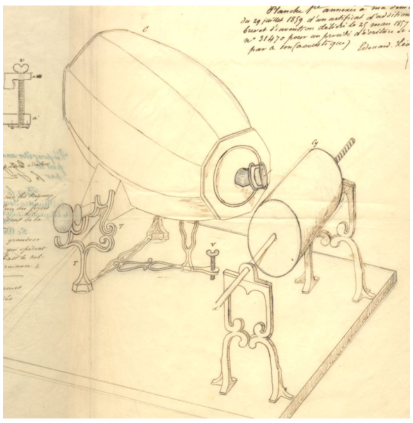
The investigations about micro-intervals will probably lead to new conceptions for *Tonnetz* in the future. But it is yet possible to feel the limits of what a spatial projection of music can really represent of its complexity.

5. Representing time.

The central problem in representing music, despite all the former considerations, remains the representation of time. The terms of this problem are quite simple: means of representation are geometric and spatial, while time is “something else”. This only fact could disqualify the idea of representation itself, has a proof of its inherent falsification of the reality. But representation is not a duplication of the reality, but a mean to understand it and to allow its replication, for memory or anticipation. The first example to be given here is the known representation of sound itself, that was invented in the nineteenth century to reproduce the voice and music by mechanical means. In fact, as we know it since the recent rediscovery of his work, the very first intent to represent the behavior of the sound was made by Édouard-Léon Scott de Martinville during the 1850s and 1860s, and it is interesting here to quote his own description of his work.

> Il s’agissait, conformément à ces principes de construire un appareil qui reproduisît par un tracé graphique les détails les plus délicats du mouvement des ondes sonores. Je devais arriver ensuite, par le secours de moyens mathématiques, à déchiffrer cette sténographie naturelle.[34]

This statement, “originally deposited in a sealed packet on 26 January 1857”, is really interesting for its definition of the representation of the sound as “a graphic trace of the most delicate details of the motion of the sound waves”. It is worth noting that the expression he uses: “natural stenography” is quite interesting also, as it deals with the idea of a notation that would be the work of nature itself (and not of a cultural convention of human beings), and that nature can be deciphered only “with the help of mathematical means”, witch is so similar to the expression used by Rameau in his *Traité de l’harmonie réduite à ses principes naturels* (1722),[35] and underlines the Cartesians founding of those considerations.



[34] “It was a matter of constructing, in accordance with these principles, an apparatus that would reproduce by a graphic trace the most delicate details of the motion of the sound waves. I had then to manage, with the help of mathematical means, to decipher this natural stenography.” Trad. Patrick Feaster in *First sounds*, December 2009.
http://www.firstsounds.org/publications/articles/Phonautographic-Manuscripts.pdf

[35] “ce principe ne peut guère nous être connu sans le secours des mathématiques”
http://gallica.bnf.fr/ark:/12148/btv1b86232459/f15.image.r=.langFR

Fig. 38: (a) Original drawing of the Phonautographe by Édouard-Léon Scott de Martinville and (b) one of the very first waveform of the history

"Writing" the sound itself seems to give a good idea of how the time can be represented. In fact, drawing a waveform is the perfect example of the relation in between time and motion, a concept dating back to Aristote, with is idea that time is « motion which can be enumerated ». In Édouard-Léon Scott de Martinville's device, the regular circular movement of the cylinder is the absolute condition of a correct rendering of the "trace", and of the adequacy of a time axis. The representation of time is not separated from the idea of its measurement. The great French scientist Henri Poincaré wrote in 1898 a decisive paper on the measurement of time: *"la mesure du temps"*.[36] He makes in this work some very interesting statements, concerning both the philosophy of time and practical considerations on physicist's difficulties to understand the nature of temporality, in its relation to space.

> *L'ordre dans lequel nous rangeons les phénomènes conscients ne comporte aucun arbitraire. Il nous est imposé et nous n'y pouvons rien changer.*
>
> *Je n'ai qu'une observation à ajouter. Pour qu'un ensemble de sensations soit devenu un souvenir susceptible d'être classé dans le temps, il faut qu'il ait cessé d'être actuel, que nous ayons perdu le sens de son infinie complexité, sans quoi il serait resté actuel. Il faut qu'il ait pour ainsi dire cristallisé autour d'un centre d'associations d'idées qui sera comme une sorte d'étiquette. Ce n'est que quand ils auront ainsi perdu toute vie que nous pourrons classer nos souvenirs dans le temps, comme un botaniste range dans son herbier des fleurs desséchées.*
>
> *Mais ces étiquettes ne peuvent être qu'en nombre fini. À ce compte, le temps psychologique serait discontinu. D'où vient ce sentiment qu'entre deux instants quelconques il y a d'autres instants? Nous classons nos souvenirs dans le temps, mais nous savons qu'il reste des cases vides. Comment cela se pourrait-il si le temps n'était une forme préexistant dans notre esprit ? Comment saurions-nous qu'il y a des cases vides, si ces cases ne nous étaient révélées que par leur contenu ?*[37]

The divergence in between psychological time and physical time was already highlighted by Henri Bergson, in his essay on the immediate data of consciousness,[38] but the remarks of Poincaré addresses Bergson's concept of "*durée*" (duration), giving another description of the reality of what is necessary for the stream of consciousness to have a

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[36] Poincaré's works on line : http://ahp-poincare-biblio.univ-lorraine.fr/?t
Henri Poincaré, "La mesure du temps", *Revue de métaphysique et de morale*, 6, numéro de janvier 1898 (Flammarion, Paris). http://www.univ-nancy2.fr/poincare/bhp/pdf/hp1898rm.pdf

[37] "The order in which we rank conscious phenomena has no arbitrary. It is imposed on us and we can not do anything to change. I have just one comment to add. If an aggregate of sensations becomes a memory that can be classified in time, it must have ceased to be in actual time, we must have lost the feeling of its infinite complexity, otherwise it would have remained present. It must have so to speak crystallized around a center of associations of ideas which will be a kind of label. Only when they have lost all life we can classify our memories in time, as a botanist arranges in his herbarium of dried flowers. But these labels can only be finite in number. If it was so, psychological time would be discontinuous. Where does this feeling that between two instants are other moments comes from? We classify our memories in time, but we know that there are blanks. How could that be if time was not a pre-existing form in our mind? How would we know that there are empty boxes, if these boxes where known only by their content?" Henri Poincaré, "La mesure du temps", *op. cit.*, p. 1-2.

[38] Henri Bergson, *Essai sur les données immédiates de la conscience*, Alcan, Paris, 1889.

clear representation of time order. Poincaré is in fact describing the structuring of time, or perhaps what philosophers call its "synthesis". The measurement of time needs to define an "interval of time" that is always referred to some spatial characteristics. It is because something is different in another space, that time can exist. This is quite different from the autonomy of the description of the ordinary space. A pure time axis does not make sense.

But there is another point highlighted by Poincaré. And this point is related to the fact that there is no time conceivable without some difference in the space. That's what is called a change of state (state or, as we will see further on, a "phase"). Poincaré first work was about dynamical systems, that is to say the part of physics that works on movements, and that allow to escape Zeno's paradoxes. His doctoral thesis is about differential equations, and that's probably why he proposes this unusual definition of time:

> *Le temps doit être défini de telle façon que les équations de la mécanique soient aussi simples que possible.*[39]

This is a quite different conception of what comes first in the definition of time, and this has very much to do with one of Poincaré's most powerful discovery, in the course of his investigations on the three-body problem and in the continuation of the work by Lagrange and Hamilton.

Il pourra se faire qu'on ait entre les $2n$ quantités x et x', une **relation:**

$$F_1(x_1, x_2, \ldots, x_n, x'_1, x'_2, \ldots, x'_n) = \text{const.}$$

Fig. 39: The first mention of a Phase Space in the work of Poincaré.[40]

Some lines after this formula, Poincaré writes :

> Quand on se donne la position initiale d'un point mobile et les équations différentielles qui définissent la loi de son mouvement, la position du point à un instant quelconque se trouve entièrement déterminée.[41]

Adding to the usual space of the positions $(x_1, x_2, \ldots, x_n)$ the space of the transitions $(x'_1, x'_2, \ldots, x'_n)$ (with $x'_1 = dx_1/dt$ etc. i.e. the celerity, the way the position is "moving"), defines another kind of space, with 2n coordinates $(x_1, x_2, \ldots, x_n, x'_1, x'_2, \ldots, x'_n)$ where a point means a "state" or a "phase". Poincaré makes it very clear that in this space, a given system should show absolute determinism, and that there is in fact no need of a time axis

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[39] "Time is to be defined in such a way that Mechanic's equations are as simple as possible." Henri Poincaré, "La mesure du temps", *op. cit.*, p. 6.

[40] "Sur le problème des trois corps et les équations de la dynamique", Poincaré Henri, *Acta Mathematica*, 13, 1890, p. 1-270 (p. 53). http://www.univ-nancy2.fr/poincare/bhp/pdf/hp1890am.pdf

[41] "When we take the initial position of a moving point and the differential equations that define the law of its motion, the point position at any instant is completely determined." *Ibid*.

to know perfectly both the past and the future. The work on the phase spaces will conduct to statistical mechanics[42] and to the chaos theory.

The important point here, is that according to those theoretical considerations, it is possible to represent the time without mentioning it directly on an axis. We will give in the following pages an example of the interest of such a conception, dealing with the analysis of melody, or to be precise, real world melody, that is to say melody not reduced to a skeletal scheme.

Within the representations that have a great influence on music and musicology in the course of the twentieth century, the sonogram is probably one of the most important, has it was a representation of the sound itself not at the microscopic level of the waveform, but at the more integrated level of the frequencies. It is very important to notice that the Fourier transform, which can be deduced from the waveform itself, adds a new dimension (in the mathematical meaning) to the representation of the sound. The structure of a sonogram is what I used to call a presence matrix. It creates a category for each frequency and determines the amount of “presence” of this frequency, in terms of energy, at each moment. The category “frequency” is the kind of category “as simple as possible”, according to Poincaré’s expression. But in fact, the sonogram is a compromise that leads to consider a short amount of time as if it was infinite, Fourier’s theorem being mathematically exact only on that condition. In fact, the way this approximation is conducted (and thus the category “frequency” defined) leads to not so slight variations.

It is unclear whether the category “frequency”, as defined for the sonogram, is implemented for our perception, but it is clear that the sonogram, the representation itself, has changed our way of perceiving the sound, in the course of the twentieth century, at least as much as the invention of musical notation has changed the way of conceiving music in its time. We are not really able to segregate a single harmonic within the audition of a note, like the sonogram can do it. Our categories for sound are quite different, and we are able to adapt them with some specific training. We can recognize really complex objects in the musical flow, and segregate them when computer algorithms have sometime difficulties in following a single note. Part of this problem is probably due to a wrong conception in the way time has to fit the variation of sound object’s temporal dimensions.

The idea of “presence matrix” can be extended to categories far more complex than single frequencies. Musical knowledge is something a little bit more sophisticated than a mere list of sinusoids. It can involve, even in the field of computer science, a big data

[42] http://archive.org/download/elementaryprinci00gibbrich/elementaryprinci00gibbrich.pdf

bank of sounds, like the ones used for Computer Assisted Orchestration.[43] But it can also concern elements at a much higher level of structure, like melodies, themes, etc. A presence matrix tells *when* the elements considered has categories are present. It is, so to speak, an "ontological" transcription. But a basic perceptual evidence must be taken into consideration: we are able to mask even very high levels of sound if they are uninteresting for us and our awareness is much higher when something changes. So the concept of "presence" is to be considered very cautiously. In fact, every presence matrix has a dynamic counterpart, what could be called a "change" matrix. One can be mathematically defined from the other using the derivation process, that is to say the function that tells how a function is changing. In the case of the sonogram, this derivation process is very simple to calculate, being the difference of energy, in a given frequency band, in between a given moment and the following one. This leads to what I called Differential Fourier Transform.[44]

Let's have a look to a simple example: the transition in between two sounds (fig. 40). In the first part of the figure, we have a classical sonogram, representing with grey levels the amount of energy at each instant, for each frequency band. In the second part, we can see the differential representation of the same, showing with red levels the augmentation of energy, and in blue levels its diminution. The first one tells the presence, the second one what is changing. When the second sound begins "flat", that is to say without vibrato, it is worth noting that the second representation is completely white, telling that at this moment, nothing is changing. If the sound was not modulated by a vibrato, this representation would have remained empty, and it is a good indication of the purpose of vibrato, allowing the sound to be not only there, but also alive.

[43] Grégoire Carpentier : « Computational Approach of Musical Orchestration Constrained Multiobjective Optimization of Sound Combinations in Large Instrument Sample Databases », Ph.D. Thesis. Supervisors: Emmanuel Saint-James (LIP6-CNRS) and Gérard Assayag (IRCAM), Tuesday, December 16, 2008.

[44] Jean-Marc Chouvel, Jean Bresson, Carlos Agon, « L'analyse musicale différentielle : principes, représentation et application à l'analyse de l'interprétation », EMS network, 2007. http://www.ems-network.org/spip.php?article294

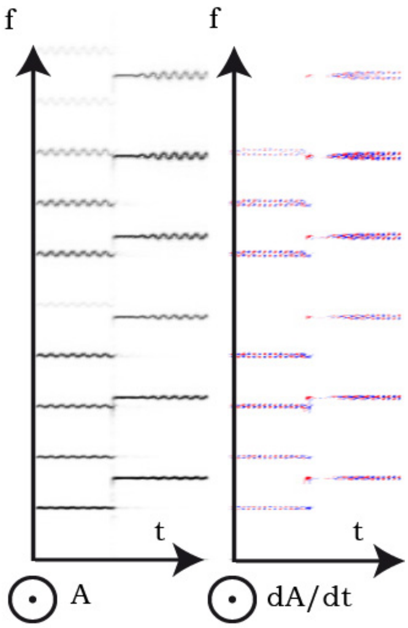
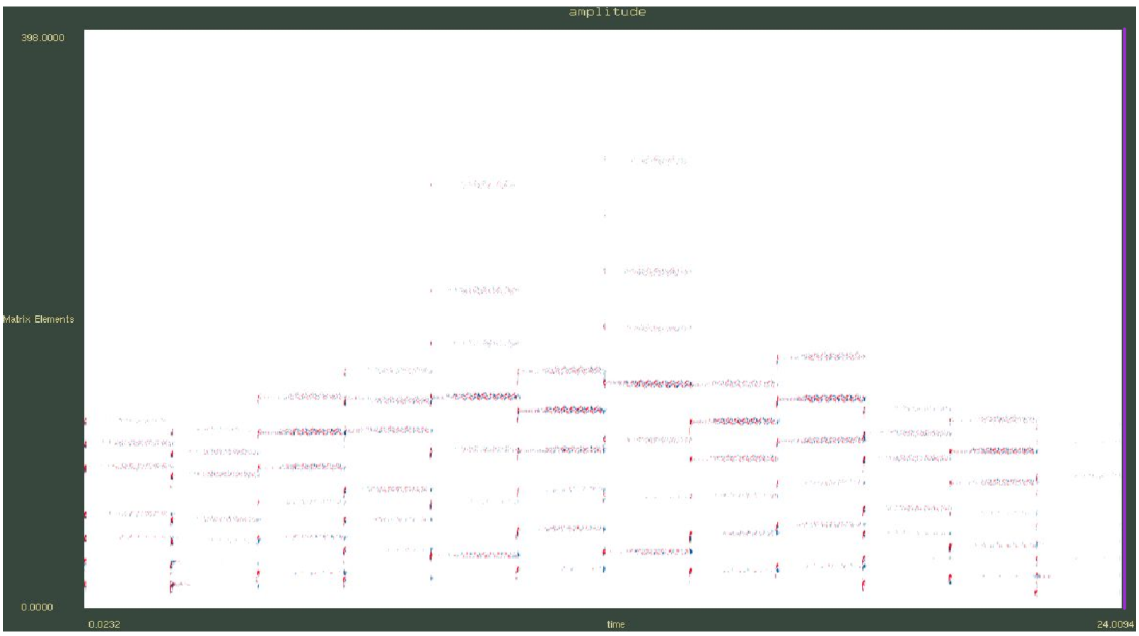


Fig. 40: Sonogram and differential sonogram of the transition in between two harmonic sounds with vibrato.

The temporal duality of musical objects is not a new idea in musical theory. It is exactly the well known duality in between note and interval. With the example of an omni-interval set, like the well-known row used by Alban Berg in his *Lyric suite*, we can understand another aspect of the importance of Differential Fourier Transform in the field of music theory.

(a)



(b)

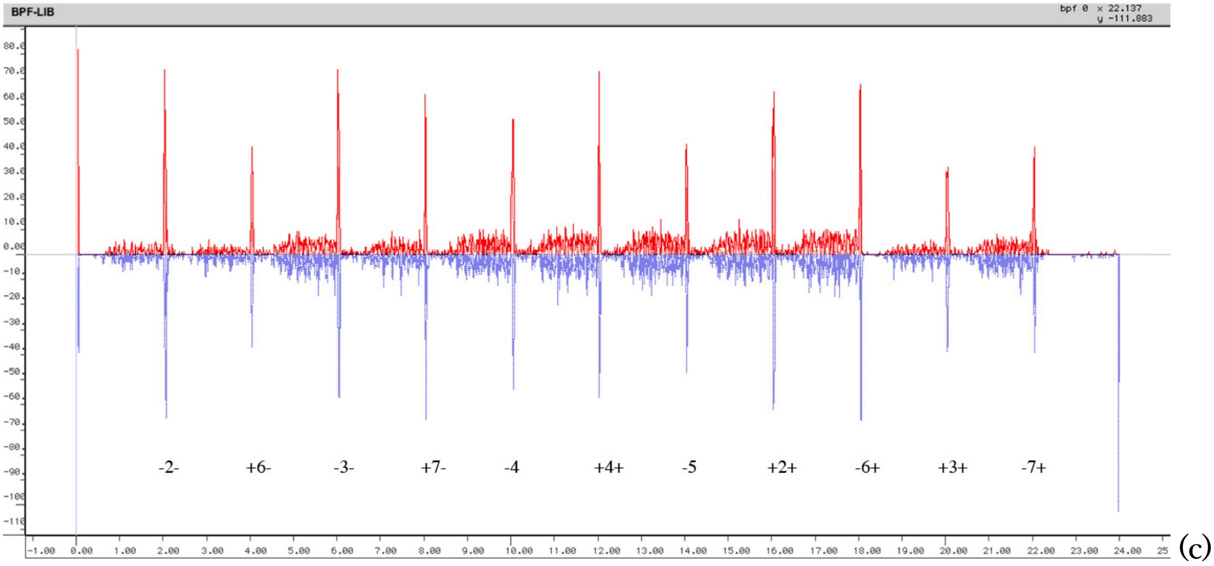


Fig. 41 : Alban Berg's row from Lyric suite (a), *it's Differential Fourier Transform (DFT)* (b) *with a synthetic musical realisation and the resulting change of energy corresponding to each interval* (c).

The kind of sound (a synthetic organ) used in this example leads to a quasi constant level (static energy), but the result is quite different if we have a look to the way this energy is "moving" ("dynamic" energy), has it is summed up in figure 41(c) where the positive (red) and negative (blue) amount of variation is represented as a whole. In this representation, that corresponds to a "profile" in two dimensions of the change matrix, the frequency information is lost, but it is possible to understand very clearly the global impact of the inner life given by the vibrato, and also, to have an idea of the difference in between the different intervals, and whether they impose a great change or insure some kind of continuity in the correspondence of their respective harmonics. Spectral continuity and discontinuity is an important component of melodic feeling, even in the mere consideration of what is signifying an interval from an acoustical point of view. This means that concordance has also a temporal signification, and much has to be done to explore what is happening when a chord follows another chord…

In the course of this survey of the importance of time for representation in music theory, a lot has been done to fade out the essentialist features and to adopt a dynamic point of view on what is the true material of this art, considering that from the microstructural waves to the macrostructural forms, moving is being. Now, it is time to ask an important question about how the "way of moving", that is therefore the "way of being", can be characterised. In other words, is there a representation for style, a way of visualising the "shape" of a behaviour? This is of course a very provocative issue, but we can try to understand it within the scope of our former reflections on dynamic systems. A piece of music is a kind of dynamic system, but it is unsure if its laws are "as simple as possible". However, it is clearly not true that in music there are no "laws", or, let's say "regularities".

The ideas developed by Poincaré in the context of complex dynamic systems and further on by Gibbs in the field of vector analysis and statistical mechanics can be useful at that point. The notion of “style” has something to do with that of predictability, hence the idea is in both cases that the phenomenon is behaving as a “system”, *i.e.* it is not completely randomized, and that the future is in some way implied by the past. Of course, there is a limit in this predictability: a work of art that would be entirely predictable might be completely boring, even if most traditional musical expressions are often ritually repetitive. We are dealing here with very complex matters, implying psychological and social involvements. But the idea is not at all to make any kind of judgement, but to be able to describe with the maximum possible accuracy the nature of the phenomenon music analysis his confronted to.

Phase diagrams are a very interesting tool, that has already been used, mostly within the field of statistical analysis. Probability is a way to tell something about time, especially if it is involved in the succession of the events. We are led to imagine that the observed temporal phenomenon can be see with two kind of “projections”: one would be what I would call a “trace”, exactly as what is done by the stylus of a phonotaugraph, and is materialised by the “time axis”, the other would be the phase space, that allows to see the phenomenon as a system, independently from its actual behaviour. The following figure can give an idea of those notions, in the very simple case of a melodic line implied in an interval step and with (a lot of) vibrato.

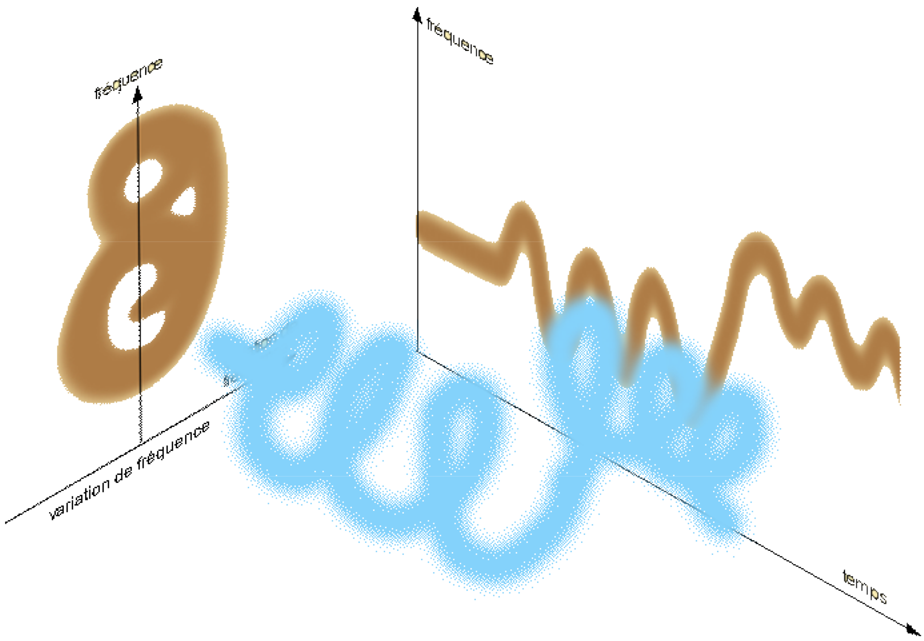


Fig. 42 : Three-dimensional representation of a melody, according to its decomposition in the following axis: frequency (f)*, frequency variation* (f’)*, and time. Left projection* {f,t} *is the “melodic line”, right projection* {f,f’} *is a “phase diagram”. Note that a third projection* {f’,t} *(not represented) is possible.*

This is only a sketch, of course, as our purpose here is to bring out the concepts. But it is clear that it gives another perception of what representing time can mean. From one single function, analysis cant bring to light new dimensions of the phenomenon. It becomes clear that this can be understood as the *purpose* of analysing: showing the dimensions of an object, such a way that it is made fully intelligible.

Conclusion: towards a general model of representation

It seems that within the course of this exploration of music representations, what has been demonstrated is the infinite variety of all the possibilities accessible to the analysts. We will now try to show that despite this apparent diversity, there is a profound unity, a general frame where all kind of representation can be inscribed. This frame is represented in next figure, in a very synthetic manner. Despite the fact our main interest has been music, this frame is very general and can apply in fact to every kind of temporal phenomenon. Time is of course a central question.

It is important to consider that the frame is organised according to a double temporal dialectic. One is what we will call “out time” and “in time” (after Xenakis concepts *“hors temps”* et *“en temps”*). A keyboard is an “out time” representation, while a score is “in time”. Both representation can be closely related : you can play a score on a keyboard. Out time representation is the conceptual frame where specific paradigms or categories[45] are listed (e.g. notes). A specific order can be related to those categories (e.g. the category “frequencies”). In fact, categories and paradigms are elements of a set, that can be structured as a space with a specific order, or not... It is also possible to minimise continuous spaces by sampling. The only condition that could be imposed for such a set to be “representative” is that it has to be a descriptor of *all* the events of the analysed phenomena. The well tempered chromatic keyboard is a bad descriptor of Arabic music, for instance. The “in time” representation is dependant on those “out time” descriptors. It is the representation of a specific work, in the words of a conceptual frame. But it can also create an order of his one, related to time, that is the successive order of the different events. If so, the “in time” representation is what I called a “formal diagram”, giving a superior legibility to the temporal strategies in a specific work.

The second temporal dialectic is in between “static” and “dynamic” representation, the second being in some way the “derivation” of the first (e.g. notes can be derived into intervals). Therefore no important difference is required for dynamic representation, has transitions can be understood, in some way, as a specific kind of categories.

There is, upon this double dialectic, a third important consideration, related to time, that we will call “structure”. Any category can be “structured” in another one, with a higher level. Basically, this is done with sets, grouping separate entities to make a

[45] Both words have

greater unity. For instance, grouping multiple frequencies to make a note, or grouping notes to make a theme. The association may be spatial or temporal (in old words harmonic or melodic), and considered on a certain number of levels. The representation is then characterised by the number of dimensions of the categories, and the number of levels of the structuration.

An important question is the relation in between categorial and temporal structures. There is often a great emphasis on this point as it is probably related to the notion of coherence, so important in the writings of composers, especially in the course of the twentieth century. Such a relation has something to do with the concept of "diagonale", and it will be therefore always interesting to make explicit the strategies involved in specific works.

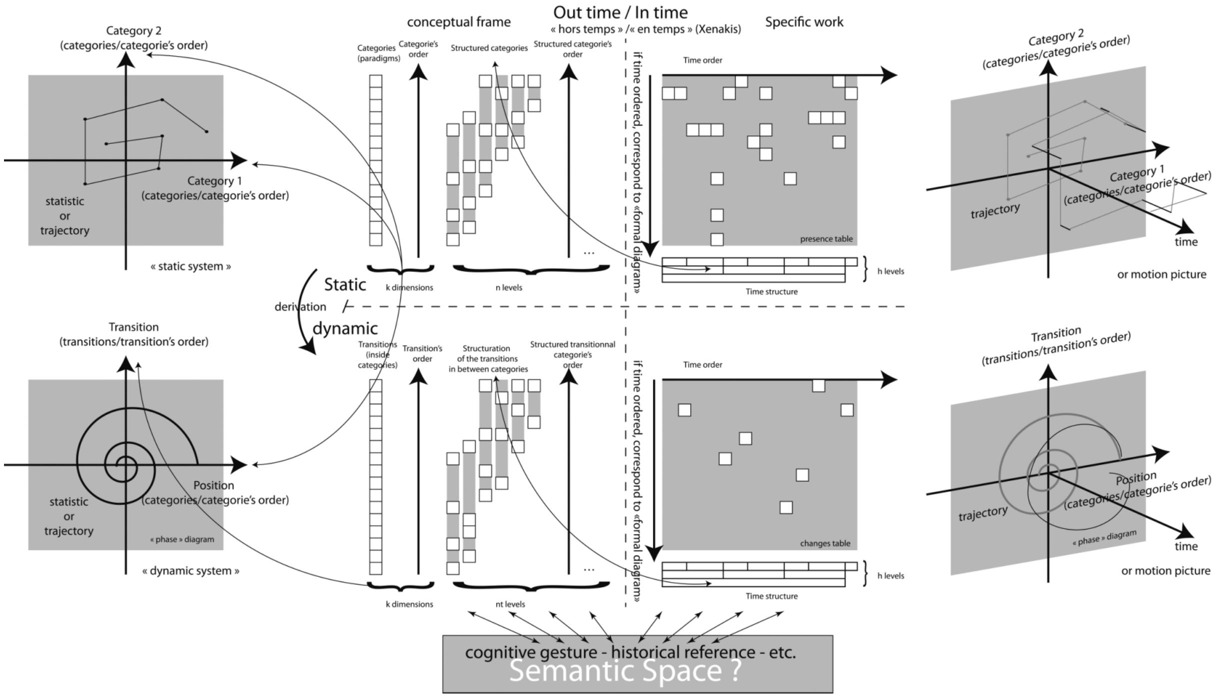


Fig. 43 : An overview of the representational possibilities for musical analysis.

From those representations arises other possibilities, for witch the same dialectics can apply. Whenever at least two independent categories can be defined it is possible to visualise their relations and the way they behave one with the other, statically, and also "in time". The expected result can be statistical or in the form of a trajectory. In both case, The static diagram can be understood as the projection according to time axis of the dynamic one. Euler's *Tonnetzs* can be a good example of this kind of diagram, with two main intervals as categories. Many expressions, like "tonal trajectory", refer implicitly to this kind of representation.

But it is also possible to mix static and dynamic categories to construct this kind of diagrams. It is of course what happens with the so-called "phase diagrams". All the "in time" versions can usefully be presented as a motion picture. Of course this kind of representation is impossible with the limitation of paper printing, but it has already appeared with multimedia supports like, for instance, the on-line review *Musimediane*.

It is important to understand the global possibilities for music representation. Now representation is not the aim of music analysis, but an important mean for its investigations. Several major methodological issues need to be addressed. First of all is perhaps the question of categories. Why use a particular category? "Tell me your categories, I'll tell you what kind of analysist you are". The preconception of the categories can sometimes be a tramp, as we tried to demonstrate with the example of the *Tonnetz*. But isn't it possible to let each piece of music define it's own categories, has it is possible to let it define the temporal order of the events? What would that mean exactly? Perhaps only to be "à l'écoute de l'œuvre", listening to the work, attentive to its characteristics, before encrypting it in a non-adapted way. It is the last that can be ask from a musicologist, to be listening… But are we able to listen with another point of departure than our previous knowledge?

A second issue is then: why choosing a representation and not another one? A compulsive mania for a specific kind of topic can have it's legitimacy, but it can also be more informative about the musicologist than about the work itself. This is not so different from the first issue. But there is another point, related to the adaptation of a particular representation to bring out a clear coherence of some specific kind of relations. That's what we tried to illustrate with the example of the different kind of *Tonnetz*. The question is then, how to select this specific representation? The attraction of an outstanding figure, a kind of "emergence" phenomenon, is always a kind of accomplishment but deserves to be questioned.

The third issue is around the *aim* of analysis : we said something like "make fully intelligible". A description, even perfect, is not an interpretation. All the representations we presented can be profoundly meaningful. But it is important to understand how. A sonogram is impossible to understand without some idea of its theoretical foundations. It is exactly the same with all the other representations. That's why I added to the former figure what I called a "semantic space". But even if it is presented as a separated layout, involving external considerations like cognitive gesture of historical references, it is fundamental to understand that it is related to every aspect of the phenomenon with two ways arrows. Meaning arises always as an inscription in the reality and in its constraints, even if we have the feeling it is independent. And it is perhaps because of this feeling that we are often unable to evidence the true nature of our relation to artistic facts (and perhaps many other kind of facts…). The representation is not, then, a false duplication of the reality, but a true possibility of appropriating it. It is a sideway that is also a shortcut. It is, in the noble meaning, a *metaphor*. Making explicit the relations, it helps the mind to objectify its subjectivity. Mersmann's propositions, we discus at the beginning, or Brower's assumptions, are attempts clearly related to this. One can find it too schematic or too simple, but that is what understanding means. The representation is what brings to the surface what was hidden under the surface. Refining its means means refining the meaning itself.