# HARMONIC CONCORDANCE

## TOWARDS A NEW APPROACH OF CONSONANCE

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# Introduction.

Consonance appears since the origins as the central notion of any musical theory, and its implication in musical composition is fundamental at least since the apparition of western polyphony. From the intuitive evidence to the theoretic statement, its formulations changed a lot throughout history, but its relevance was never truly questioned. However, if the mains poles of the notion lead to general agreement, dissension begins when "intermediate zones" are put to discussion. Unfortunately, musical reality mainly consist of what happens in those "intermediate zones". Serge Gut<sup>1</sup>, after trying to summarise the steps and tendencies of this process from the ancients to nowadays, is led to the conclusion that "the problems stated by this phenomena are infinitely vast and complex, and far from being all solved nor even approached".

This paper intends to come back to the acoustic foundations of harmony, so that a coherent epistemological construction can be reached at the different levels of the notion's structure<sup>2</sup>. What follows must be understood not as the development of one of the hypothesis listed in Gut's paper, the so-called "common harmonics" theory, but as the construction, from a truly physical point of view, of a new notion - concordance - on behalf of which we will try to show how it leads to a comprehensive tool to understand all the existing former theory, and how, when it gives back to the phenomenon its original complexity, the conceptual unicity of the consonance-dissonance polarisation needs to be overstepped. The following sketch gives the direction of the methodological process that will be ours.

<sup>&</sup>lt;sup>1</sup>Serge Gut: « Consonance, dissonance » in *Science de la musique*, Marc Honegger dir., Bordas, Paris, 1976, t.1 pp. 245-249. <sup>2</sup>In a former paper about *Physics and Aesthetics* (Chouvel 95), we developed the idea that a confusion in the epistemological levels was conducting to certain lack of differentiation prejudicial to the understanding of the phenomena.



Fig. 1 : Sketch of the epistemological process.

# I. Essay on the physical theory of harmony.

• SUPERPOSITION OF TWO WAVE FORMS :

We will use in this chapter the notations of signal processing<sup>3</sup> derived from Dirac's quantum mechanic.

Harmonic phenomenon is related to the superposition of two temporal signals  $X_1(t) = \langle t | X_1 \rangle$  and  $X_2(t) = \langle t | X_2 \rangle$ . This superposition, if it is realised in the three dimensions space with "ponctual" sources, leads to interference figures. To avoid the spatial complexity of the phenomenon, we can limit our investigation to a ponctual superposition<sup>4</sup> in linearity conditions. We simply observe then a resulting signal as  $X(t) = X_1(t) + X_2(t)$ .

• ENERGETIC ANALYSIS :

The energy of such a signal is :

$$\|X_1 + X_2\|^2 = \|X_1\|^2 + \|X_2\|^2 + 2\langle X_1, X_2 \rangle$$
(1)

 $\|X_1\|^2$  being the energy of the first signal;  $\|X_2\|^2$  that of the second one;  $2\langle X_1, X_2 \rangle$  represent the energy related to the interaction (or *coupling term*). It is this term that will allow us to define what we will call the *harmonic concordance*. But we should first notice that the simple operation of signal superposition leads to paradoxical results. This can even lead to the total absorption of both signal's

<sup>&</sup>lt;sup>3</sup>For more precision about this notation, cf. Georges BONNET : *Considérations sur la représentation et l'analyse harmonique des signaux déterministes ou aléatoire* in *Théories et techniques de la détection en acoustique sous-marine et traitement du signal*, Masson & Cie, Paris, colloque de Nice 17-20 Avril 1967. See also LAI (D.C.) : *Signal space concepts and Dirac's notation*, John Hopkins Univ. Rept. N° AFCRC-TN-60-167, 1960, and of course DIRAC (P.A.M.) : *Quantum Mechanics*, Clarendon, Oxford, 1958, in an other context.

<sup>&</sup>lt;sup>4</sup>e.g. what can be realised electronically with a monophonic generator. Note that this restriction has no incidence on the essential properties of harmony, all the interference phenomena being considered as a side effect very rare in standard musical situation.

energy<sup>5</sup>. The so called resulting sounds are related to the fluctuation of the energy in the case of a slight detune in between the signal's frequencies. The existence of such phenomena, even if they obviously can be taken into account by the coupling term, makes it difficult to understand harmony from the temporal expressions.

### • FREQUENTIAL INTERPRETATION :

According to Dirac's notation, we can also consider the frequential projections :  $X_1(v) = \langle v | X_1 \rangle$  and  $X_2(v) = \langle v | X_2 \rangle$ . The energy of the resulting signal is given by the following expression :

$$\|X_1 + X_2\|^2 = \int_{v} (X_1(v) + X_2(v))^2 dv$$
  
=  $\int_{v} (X_1(v))^2 dv + \int_{v} (X_2(v))^2 dv + 2\int_{v} X_1(v) \cdot X_2(v) dv$ 

Concordance is then defined as the *coupling term* :

$$C(X_1, X_2) = \int_{v} X_1(v) \cdot X_2(v) dv.$$
 (2)

Note that if v is real and >0, phase effects are not taken into account. Furthermore, if the former expression gives no specifications for the spectral composition of the sounds, those sounds are however supposed to have the sufficient duration and stability for the frequential analysis to be performed. In the usual field of musical harmony, those remarks have no incidence.

#### • SPECIFIC CASE OF HARMONIC SOUNDS :

Harmonic sounds are more directly concerned by the consonance and dissonance notions, as most instrumental sounds involved in the harmony phenomena belong to this category. We will not limit ourselves to infinite continuous sounds, according to a mathematical point of view, but on the contrary we will try to work with realistic sounds with a straight envelope and sufficient duration just as most orchestral instruments and organ can produce. Those sounds can be modelled this way :

$$X(I) = \sum_{k \in N^*} a_k G(I_k - I)$$

$$I = \log_2 \left(\frac{f}{f_{ref}}\right); I \text{ interval in octaves;}$$
(3)

<sup>&</sup>lt;sup>5</sup>The simplest way to convince oneself of this paradoxical statement is to consider two signals as  $X_1(t) = -X_2(t) \neq 0$ . It is clear that the resulting signal has no energy.

$$\begin{split} &I_{k} = \log_{2} \left( \frac{k \cdot f_{0}}{f_{ref}} \right); I_{k} \text{ intervals of the spectra in octaves;} \\ &f_{0} \text{ fundamental frequency; } f_{ref} \text{ reference frequency (1Hz);} \\ &a_{k} \text{ weight of the harmonic components;} \\ &G \text{ indeterminacy function, in further calculations G will be a Gaussian:} \\ &G(I_{k} - I) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2\sigma^{2}}(I - I_{k})^{2}}; \text{ where } \sigma \text{ is the standard deviation.} \end{split}$$

This model can be approached in several ways, as the spectral indeterminacy may occur at any level of the information channel that leads from the instrumental production of the sound to the psychoacoustics mechanisms involved in the listener's ears. On a theoretical point of view, it is to be noticed that there are two main ways of conceiving this indeterminacy, both leading anyway to the same former expression.

A first one would take into account the spectral distribution of the energy : the concordance would then represent the energy shared by the two signals, leading to a spectral fusion. This interpretation in the field of energy is to be confronted to the theory of Helmoltz's resonators. The fraction of energy received by a resonator compared to the maximum energy received at the resonance is :

$$\frac{W}{W} = \frac{1}{\left(\frac{M}{2\alpha}\right)^2 + 1}$$

 $\alpha = \frac{h}{k}$ ; h being the amortising factor and k the resonator's own

pulsation,

M =  $i - \frac{1}{i} \approx 2 \log i$  with  $i = \frac{\omega}{k}$ ,  $\omega$  excitation pulsation.

According to our notations,  $I_1 - I_2 = \log i$ .

Then: 
$$\frac{W}{W} = \frac{1}{\left(\frac{I_1 - I_2}{\alpha}\right)^2 + 1} \approx 1 - \left(\frac{I_1 - I_2}{\alpha}\right)^2 \approx e^{-\frac{1}{\alpha^2}(I_1 - I_2)^2};$$

Second order equivalence is obtained if  $\alpha = \sqrt{2}\sigma$ .

Concordance can also be understood as the product of the presence probability of  $X_1$  et  $X_2$  on the  $\nu$  frequency (distribution theory), that is to say the probability of a coincidence of both signals on this frequency.

Whatever the paper of G, the importance of  $\sigma$  causes no doubt. Several interpretations of this term are possible throughout the semiotic channel conducting from the acoustic phenomena to the listener's interpretation. One as to be conscious that there is not a definite answer to the localisation of  $\sigma$  in one of the element of this channel, mainly because it depends on the precise conditions of the musical event. An orchestral tutti, a voice with vibrato, and an electronic organ does not have the same initial indeterminacy. On the other hand, human ear is certainly not an absolute precision receptor.



Fig. 2 : Basic levels in which the indeterminacy factor  $\sigma$  can be involved as a kind of filtering, together with the possible interferences of perceptive and cultural expectations and conditioning.

On the acoustic level, the analysis of instrumental spectrums shows that the proposed model is a good approximation of their real shape. The behaviour of the ear is more complicated to describe. A mechanist model consisting of a great number of resonators with graduated tuning and amortising factor has often been proposed and could be coherent with the following calculations. We will try to give in the second chapter of this study some complements about the psychoacoustic prolongation of concordance.

• CASE OF TWO PARTIALS :

Concordance, in this case, describes the "spectral fusion" phenomena.

$$X_{1}(I) = a_{1}G(I_{1} - I); X_{2}(I) = a_{2}G(I_{2} - I);$$

$$C(X_{1}, X_{2}) = \int_{I} X_{1}(I) \cdot X_{2}(I) dI = \frac{a_{1}a_{2}}{(\sigma\sqrt{2\pi})} \int_{I} e^{-\frac{1}{2\sigma^{2}}\left(\left(I - I_{1}\right)^{2} + \left(I - I_{2}\right)^{2}\right)} dI$$

$$As: (I - I_{1})^{2} + (I - I_{2})^{2} = \frac{1}{2}(I_{1} - I_{2})^{2} + 2\left(I - \frac{I_{1} + I_{2}}{2}\right)^{2} \text{ then }:$$

$$C(x_{1}, x_{2}) = \frac{a_{1}a_{2}}{(\sigma\sqrt{2\pi})^{2}} e^{-\frac{1}{2\sigma^{2}}\frac{(I_{1} - I_{2})^{2}}{2}} \int_{I} e^{-\frac{1}{\sigma^{2}}\left(I - \frac{(I_{1} + I_{2})}{2}\right)^{2}} dI = \frac{a_{1}a_{2}}{\sigma\sqrt{2}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\sigma\sqrt{2\pi}\right)^{2}}$$
(5)

With  $\sigma' = \sigma \sqrt{2}$  the former expression becomes a normalised function. The following graphics explain the principle of this calculation.



*Fig. 3* : The concordance of two partials is represented by the surface under the curve obtained by multiplying the curves of both partials. This mesures how much the sounds have been confused in the superposition.

• CASE OF TWO HARMONIC SOUNDS :

With 
$$X_1 = \sum_{k \in \mathbb{N}^*} x_{1,k}$$
 and  $X_2 = \sum_{n \in \mathbb{N}^*} x_{2,n}$ , the former result can be

generalised :

$$C(X_{1},X_{2}) = \sum_{(k,n)\in\mathbb{N}^{*^{2}}} C(x_{1,k},x_{2,n}) = \sum_{(k,n)\in\mathbb{N}^{*^{2}}} \frac{a_{1,k}a_{2,n}}{\sigma'\sqrt{2\pi}} e^{-\frac{(I_{k}-I_{n})^{2}}{2\sigma'^{2}}}$$

It is obviously easier to represent C(I) that is to say the concordance of a variable sound  $(I_2 = I)$  with a similar fixed one  $(I_1$  being the origine). The following formula gives the two dimensional representations of figure 4 and 5.

$$C(I) = \sum_{(k,n) \in N^{*2}} \frac{a_{1,k} a_{2,n}}{\sigma' \sqrt{2\pi}} e^{-\frac{(\log(k/n)/\log 2 - I)^2}{2\sigma'^2}}$$
(6)

The main parameters involved in the calculation of the concordance are the  $a_k$  series specifying the timbre and the uncertainty  $\sigma$ . Figure 4 illustrates the importance of the successive harmonics for a given value of  $\sigma$ .



Fig. 4 : C(I) for  $\sigma=0,002oct$  and different values of n.

In figure 5, we chose, with a timbre giving an uniform decreasing paper to the successive harmonics, to show more specifically the influence of  $\sigma$ .



Fig. 5: Harmonic discordance, case of two notes. C(I) for I from 0 to 1 octave ( $D = -10 \log_{10} C$ ), The decreasing factor for  $a_{1,k}$  being around -12dB/o and  $\sigma$  taking the following values (from the top to the bottom) 0,002/0,006/0,010/0,014

# **II.** Concordance and perception.

### • TOWARDS A PSYCHOACOUSTICAL INTERPRETATION :

When musical production, for example in the case of a synthesiser, allows very minimal values for the spectral precision  $\sigma$ , the feeling of harmony remains equivalent. The ear gives a limitation to the precision that can be expected from the whole semiotic channel. This can be understood on the well-known diagram reported by Stevens (1938):



Fig. 6 : Map of the audible domain in the plane : sound pressure level versus frequency of sine waves. The closed curves indicate the level of acoustical information : number of quantified squares dLdf per unit of surface. (after Stevens 1938)

Even if this diagram mixes frequency and dynamic resolution, it helps us with two main ideas. First of them is the fact that the resolution increases<sup>6</sup> with the fundamental frequency and the sound pressure level. This means that a chord will fusion better if the level is low (result that agrees with common experience in chamber and orchestral music), and that the concordance will not be identical for the same chord in high and low register position. It is also to be noticed that, according to this diagram, the resolution may approximately be the same for all spectral components of a note, with the hypothesis of a constant logarithmic decay of about -15dB/oct.

<sup>&</sup>lt;sup>6</sup>Which means that the physiological equivalent of  $\sigma$  decreases...

This second idea allows us to propose a first approach of the influence of psychoacoustical data in the field of harmonic concordance. We can not pretend to give a definitive model when there is no certainty about the real meaning of the  $\sigma$ parameter in the case of the ear, and even less about its effective values. And what about the individual differences in between listeners? Anyway we will not give too much importance to those considerations, and the purpose of the present chapter is more to give an idea of the tendencies than to solve definitively the problem.

### • GENERALISATION OF THE CONCORDANCE DEFINITION :

The concordance of two partials is now to be defined with different values for  $\sigma$ . This is nothing but a generalisation of the former results.

$$X_{1}(I) = a_{1}G_{1}(I_{1} - I), \text{ with } G_{1}(I_{k} - I) = \frac{1}{\sigma_{1}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{1}^{2}}(I - I_{k})^{2}};$$

$$X_{2}(I) = a_{2}G_{2}(I_{2} - I), \text{ with } G_{2}(I_{k} - I) = \frac{1}{\sigma_{2}\sqrt{2\pi}}e^{-\frac{1}{2\sigma_{2}^{2}}(I - I_{k})^{2}};$$

$$C(X_{1}, X_{2}) = \int_{I}X_{1}(I).X_{2}(I)dI = \frac{a_{1}a_{2}}{\sigma_{1}\sigma_{2}2\pi}\int_{I}e^{-\frac{1}{2\sigma_{1}^{2}}(I - I_{1})^{2} - \frac{1}{2\sigma_{2}^{2}}(I - I_{2})^{2}}dI;$$

$$\left(\frac{I - I_{1}}{\sigma_{1}}\right)^{2} + \left(\frac{I - I_{2}}{\sigma_{2}}\right)^{2} = \left(\sigma_{1}^{2} + \sigma_{2}^{2}\right)\left(I - \frac{\sigma_{1}^{2}I_{2} + \sigma_{2}^{2}I_{1}}{\sigma_{1}^{2} + \sigma_{2}^{2}}\right)^{2} + \frac{\sigma_{1}^{2}\sigma_{2}^{2}}{\sigma_{1}^{2} + \sigma_{2}^{2}}(I_{1} - I_{2})^{2} \text{ and then }:$$

$$C(x_{1}, x_{2}) = \frac{a_{1}a_{2}}{\sqrt{\sigma_{1}^{2} + \sigma_{2}^{2}}\sqrt{2\pi}}e^{-\frac{1}{2\sqrt{\sigma_{1}^{2} + \sigma_{2}^{2}}(I_{1} - I_{2})^{2}}$$

$$(7)$$

With  $\sigma'' = \sqrt{\sigma_1^2 + \sigma_2^2}$  the former expression becomes a normalised function. In the case of two harmonic sounds, according with the hypothesis of a constant value of  $\sigma$  for all the harmonics of a same note, we are led to the following expressions :

$$C(X_1, X_2) = \sum_{(k,n)\in\mathbb{N}^{*2}} C(x_{1,k}, x_{2,n}) = \sum_{(k,n)\in\mathbb{N}^{*2}} \frac{a_{1,k}a_{2,n}}{\sigma"\sqrt{2\pi}} e^{-\frac{(I_k - I_n)^2}{2\sigma"^2}}$$

 $C(I) \text{ is the concordance of a variable sound } (I_2 = I) \text{ with a similar}^7 \text{ fixed} one (I_1 \text{ being the origin}). \sigma'' can be consider as a function of the interval. If <math>\sigma_1$  is the fixed central value,  $\sigma_2$  can be approximated as a linear function of  $\sigma_1$  and I:

<sup>&</sup>lt;sup>7</sup>similar means here exclusively " that have the same timbre ", i.e. the same values for the partial's coefficients.

 $\sigma_2 = \sigma_1(1 - pI)$  and then :  $\sigma'' = \sigma_1 \sqrt{(1 + (1 - pI)^2)}$ , p being the slope, it is to say the degree of

variation.

(6) becomes then :

$$C(I) = \sum_{(k,n) \in \mathbb{N}^{*2}} \frac{a_{1,k}a_{2,n}}{\sigma_1 \sqrt{\left(1 + (1 - pI)^2\right)} \sqrt{2\pi}} \cdot e^{-\frac{\left(\log(k/n)/\log 2 - I\right)^2}{2\sigma_1^2 \left(1 + (1 - pI)^2\right)}}$$
(8)

Of course this is only a way to have an idea on the influence of the variation of  $\sigma$ . Phychoacousticians should intent now to give some more precise values to  $\sigma$  and to modelise its variation with the frequency. The confusion about critical bandwidth (see Parncutt 89 pp. 85-86) leads to be cautious about this subject. Anyway, the definitions as well as the values given seems to bear no relationship with what is called here  $\sigma$ .

The greatest achievement of (8) compared to the "physical" results (6), is to evidence the non-symmetry of harmonic perception. For instance, why does a major and a minor chord differ so importantly when their intervallic components are the same? The dissymmetry of psychoacoustical concordance, due to the evolution of the ear precision with the frequency, gives at least a beginning of explanation.

## **III.** A new light on the notion of consonance.

• About this interval classification business :

Musicologists and music theorists, in contradiction with the common sense that compares more often harmony with colour, tried since centuries to explain harmony from the point of view of an unique axis that would have been the so-called "consonance". All those approaches, even if they coincide about the extremes as octave or semitone, remain quite uncertain about the behaviour of those intermediary intervals as tone, thirds or sixth. And it is not only a problem of binary separation in a continuum. Figure 5 resumes the principal historic data about this theme. The two first diagrams of this figure does not involve a scale but a classification, deducted from very simple considerations about the fifth's cycle and the harmonic series. Even the existence of a scale, which means in some way an unit of measure, is problematic for the notion of consonance<sup>8</sup>. It is yet uneasy to decide what is a sound *two times* louder than an other one, especially with complex sounds, and despite the apparent unidimensionnality of loudness, how can we expect to decide whether a chord, even an interval of two notes, is *two times* more dissonant than an other? The discrepancy in between the results of those diagrams is not very surprising.

For Hutchinson and Knopoff (1978), the consonance depends on the frequency range. This is consistent with the results for concordance<sup>9</sup> if a dependence of  $\sigma$  with frequency is taken into account (we will see further the nature of this relation). More generally, through the  $\sigma$  parameter, the concordance depends both on the conditions of sound production and on perceptive discrimination capacities. A broad vibrato, for example, gives little chance to a specific micro-interval harmony. The tritone is quite characteristic of the versatility of the notion of concordance. Discordance in between two peaks of concordance in a "pentatonic" context (when  $\sigma'$  is around 0,02), it becomes a concordance in between two discordances in a "chromatic" situation ( $\sigma'$  around 0,005), and again a discordance in between two concordances in an "infra-chromatic" context ( $\sigma'$  about 0,005)! This example gives a perfect illustration of the way concordance modifies our understanding of harmony (and many judgements made on false evidences) to give back to our musical feeling a certain kind of subtlety.

The last conclusion concerns more directly the data from Parncutt's psycho-acoustical theory of harmony (1989). The introduction in calculations of "standard" psychoacoustic measurements seems to lead to certain difficulty in the differentiation of the intervals, even if the *complex tonalness* is not exactly to be compared to concordance.

<sup>&</sup>lt;sup>8</sup>Consonance as usually no dimension. It is important to notice that concordance is related to energy.

<sup>&</sup>lt;sup>9</sup>The difference is that in Hutchinson and Knopoff's calculations the consonance of intervals as fifth and fourth is modified when the frequency increases. This is due to the influence of the curve chosen as critical bandwidth. It is the same in all calculations involving Helmoltz's theory : the shape of this curve is very uneasy to decide, and unfortunately has considerable influence on the aspect of the results.



Fig. 6 : Some historic proposals for a measure or a classification of intervals.

• CASE OF N NOTES CHORDS :

A generalisation can be obtained for n notes chords :

$$\begin{split} \mathbf{X} &= \sum_{n} \mathbf{X}_{n} \text{ is the signal resulting of the superposition of the notes.} \\ \|\mathbf{X}(\mathbf{I})\|^{2} &= \int_{\mathbf{I}} \left( \sum_{n} \mathbf{X}_{n}(\mathbf{I}) \right)^{2} d\mathbf{I} \\ &= \sum_{n} \|\mathbf{X}_{n}(\mathbf{I})\|^{2} + 2 \sum_{i \neq j} C(\mathbf{X}_{i}, \mathbf{X}_{j}) \\ \sum_{i \neq i} C(\mathbf{X}_{i}, \mathbf{X}_{j}) \text{ is the concordance of the n sounds.} \end{split}$$

(Remark that with normalised signals, this corresponds to the arithmetic mean of the concordance of the intervals involved in the chord.)

Many other statistic parameters can be defined, this with the aim to differentiate the chords that would result to have the same concordance (e.g. chords for which the mean of the intervals is the same when the nature of those intervals is quite different, or chords with a symmetric structure as C E G and C Eb G or as G Ab C and G B C). The standard deviation of the concordance of the intervals, what could be called the *harmonic coherence*, could represent a measure of harmonic tension's homogeneity. An other interesting parameter would be the absolute coincidence, defined as :  $D(X_1,...,X_n) = \int_{I} \prod_{k=1}^{n} X_k(I) dI$ , and that measures the amount of

energy shared by *all* the notes in the chord. All those parameters and others can help in the conception of a general harmonic typology constructed on *a priori* different basis than those of traditional western theory. Beyond a confrontation of physical, perceptual and historical data, the extension of such a typology to microtonality is of primary importance in contemporary music investigations.

# **IV. Toward an unification of the theories :**

Even if we can not here give to this subjet too much extension, it is important to understand how concordance helps us to understand nearly all the main historic hypothesis about consonance.

## • THE "BASSE FONDAMENTALE" (*RAMEAU*) :

For a given chord of n notes, it is possible to calculate its concordance with a lower note. The greater this concordance, the more redondant this lower note's spectrum with that of the chord. The use of the note that gives the best result as a fundamental bass or "root" for the chord seems to be rather logical. The calculation provided by concordance is far more realistic than that proposed by former papers on the subjet<sup>10</sup>.

## • THE INVERTED HARMONICS (*RIEMANN*) :

They represent all the notes that have in common an harmonic (the first note of the serie). It can be consider as a first, but very lacunary, approach of concordance. Anyway this is an interpretation very far from Riemann's initial idea.

## $\bullet$ Consonance, « Roughness » and Concordance :

Therhard already shown (1977) the great similitude in between roughness and consonance. The former diagrams (fig. 4) proves that there is a direct acoustic connection in between the hypothesis of quasi constant desagreement frequency (around 33Hz for Helmotz), and the evolution of the sensibility of the ear with the frequency (assimilated to  $\sigma$ ). As a first approximation, the following table gives numerical results to be compared to psychoacoustical data.

$\sigma = \sigma' / \sqrt{2}$ (octave)	σ' (octave)	Interval of maximum discordance (octave)	corresponding frequency of the resulting sound f <sub>0</sub> (octave)	f <sub>0</sub> for a resulting frequency of 33Hz (Hz)
0,0014	0,002	0,02	-6,16	2360
0,0035	0,005	0,05	-4,83	938
0,0071	0,01	0,09	-3,96	514
0,0141	0,02	0,14	-3,29	323

*Fig.* 6 : (as reference: lcent=0,00083octv; lsavart=0,0033octv; lcomma (Holder)=0,0185octv)

<sup>&</sup>lt;sup>10</sup>We refer particularly to Parncutt (1988).

Taking into account the great differences in the values proposed by experimentalists and the individual differences in between listeners, it is difficult to give a formal interpretation. However the numerical values are consistent with basic psycho-acoustical data. The acoustic nature of the relation in between this quasi constancy of the roughness zone and the evolution of the sensibility of the ear with the frequency allows an important renewal of our conceptions on the subject.

## • FORM THEORY (*CHOUVEL 90*) :

The principles of concordance definition match up the fundamental notions of form theory. First of them identity and difference, concordance being a way to measure what is redundant and what is different in the confrontation of two spectrum. Form theory leads to give also a temporal meaning to concordance, providing a way to analyse the rate of coherence and that of rupture in the succession of two chords for example. This is a broad new area of investigation in the field of harmonic analysis.

# **Conclusion.**

The fact of deducting the notion of concordance from energetic considerations gives a physical basis to the problem of consonance. On an epistemological point of view, it is interesting to notice that concordance comes directly from the "coupling term" of quantum mechanic, which is nothing but a generalisation of Pytagore's theorem... Concordance, that is in fact the scalar product of two sounds considered in a vectorial space, looks as a modern avatar of the music of the spheres. But with the numerical precision of today's computers, the physical image of the world can integrate the imprecision, and thus give an interpretation of complex phenomena, which our perception can therefore refer to. Concordance appears to be able to give to the dispersion of the concepts developed in the history about consonance and dissonance a true coherence. Giving unity to disparate notions is always satisfactory for a scientist. But this conceptual unity conducts to throw back into question the unicity of the dissonance-consonance bipolarisation and to propose a far more differentiated range of parameters to describe the harmony phenomenon. We are facing a truly perfect example of what Bachelard was calling an "opening from the inside of the notion", that is to say the transformation of the knowledge from external determinations into internal

functional structure, transformation that is, according to the French philosopher, a crucial moment in the development of epistemological consciousness.

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